



# NCEES

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## Reference Handbook

9.2 Version for Computer-Based Testing

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ISBN 978-1-932613-67-4

Printed in the United States of America  
Third printing June 2014  
Edition 9.2

# PREFACE

## **About the *Handbook***

The Fundamentals of Engineering (FE) exam is computer-based, and the *FE Reference Handbook* is the only resource material you may use during the exam. Reviewing it before exam day will help you become familiar with the charts, formulas, tables, and other reference information provided. You won't be allowed to bring your personal copy of the *Handbook* into the exam room. Instead, the computer-based exam will include a PDF version of the *Handbook* for your use. No printed copies of the *Handbook* will be allowed in the exam room.

The PDF version of the *FE Reference Handbook* that you use on exam day will be very similar to the printed version. Pages not needed to solve exam questions—such as the cover, introductory material, and exam specifications—will not be included in the PDF version. In addition, NCEES will periodically revise and update the *Handbook*, and each FE exam will be administered using the updated version.

The *FE Reference Handbook* does not contain all the information required to answer every question on the exam. Basic theories, conversions, formulas, and definitions examinees are expected to know have not been included. Special material required for the solution of a particular exam question will be included in the question itself.

## **Updates on exam content and procedures**

NCEES.org is our home on the Web. Visit us there for updates on everything exam-related, including specifications, exam-day policies, scoring, and practice tests. A PDF version of the *FE Reference Handbook* similar to the one you will use on exam day is also available there.

## **Errata**

To report errata in this book, send your correction using our chat feature on NCEES.org. We will also post errata on the Web site. Examinees are not penalized for any errors in the *Handbook* that affect an exam question.



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## UNITS

The FE exam and this handbook use both the metric system of units and the U.S. Customary System (USCS). In the USCS system of units, both force and mass are called pounds. Therefore, one must distinguish the pound-force (lbf) from the pound-mass (lbm).

The pound-force is that force which accelerates one pound-mass at  $32.174 \text{ ft/sec}^2$ . Thus,  $1 \text{ lbf} = 32.174 \text{ lbm-ft/sec}^2$ . The expression  $32.174 \text{ lbm-ft/(lbf-sec}^2)$  is designated as  $g_c$  and is used to resolve expressions involving both mass and force expressed as pounds. For instance, in writing Newton's second law, the equation would be written as  $F = ma/g_c$ , where  $F$  is in lbf,  $m$  in lbm, and  $a$  is in  $\text{ft/sec}^2$ .

Similar expressions exist for other quantities. Kinetic Energy,  $KE = mv^2/2g_c$ , with  $KE$  in (ft-lbf); Potential Energy,  $PE = mgh/g_c$ , with  $PE$  in (ft-lbf); Fluid Pressure,  $p = \rho gh/g_c$ , with  $p$  in (lbf/ft<sup>2</sup>); Specific Weight,  $SW = \rho g/g_c$ , in (lbf/ft<sup>3</sup>); Shear Stress,  $\tau = (\mu/g_c)(dv/dy)$ , with shear stress in (lbf/ft<sup>2</sup>). In all these examples,  $g_c$  should be regarded as a unit conversion factor. It is frequently not written explicitly in engineering equations. However, its use is required to produce a consistent set of units.

Note that the conversion factor  $g_c$  [ $\text{lbm-ft/(lbf-sec}^2)$ ] should not be confused with the local acceleration of gravity  $g$ , which has different units ( $\text{m/s}^2$  or  $\text{ft/sec}^2$ ) and may be either its standard value ( $9.807 \text{ m/s}^2$  or  $32.174 \text{ ft/sec}^2$ ) or some other local value.

If the problem is presented in USCS units, it may be necessary to use the constant  $g_c$  in the equation to have a consistent set of units.

METRIC PREFIXES			COMMONLY USED EQUIVALENTS	
Multiple	Prefix	Symbol		
$10^{-18}$	atto	a	1 gallon of water weighs 8.34 lbf 1 cubic foot of water weighs 62.4 lbf 1 cubic inch of mercury weighs 0.491 lbf The mass of 1 cubic meter of water is 1,000 kilograms 1 mg/L is 8.34 lbf/Mgal	TEMPERATURE CONVERSIONS
$10^{-15}$	femto	f		
$10^{-12}$	pico	p		
$10^{-9}$	nano	n		
$10^{-6}$	micro	$\mu$		
$10^{-3}$	milli	m		
$10^{-2}$	centi	c		
$10^{-1}$	deci	d		
$10^1$	deka	da		
$10^2$	hecto	h		
$10^3$	kilo	k	$^{\circ}\text{F} = 1.8 (^{\circ}\text{C}) + 32$ $^{\circ}\text{C} = (^{\circ}\text{F} - 32)/1.8$ $^{\circ}\text{R} = ^{\circ}\text{F} + 459.69$ $\text{K} = ^{\circ}\text{C} + 273.15$	
$10^6$	mega	M		
$10^9$	giga	G		
$10^{12}$	tera	T		
$10^{15}$	peta	P		
$10^{18}$	exa	E		

### IDEAL GAS CONSTANTS

The universal gas constant, designated as  $\bar{R}$  in the table below, relates pressure, volume, temperature, and number of moles of an ideal gas. When that universal constant,  $\bar{R}$ , is divided by the molecular weight of the gas, the result, often designated as  $R$ , has units of energy per degree per unit mass [ $\text{kJ/(kg}\cdot\text{K)}$  or  $\text{ft-lbf/(lbm}\cdot^{\circ}\text{R)}$ ] and becomes characteristic of the particular gas. Some disciplines, notably chemical engineering, often use the symbol  $R$  to refer to the universal gas constant  $\bar{R}$ .

### FUNDAMENTAL CONSTANTS

Quantity		Symbol	Value	Units
electron charge		$e$	$1.6022 \times 10^{-19}$	C (coulombs)
Faraday constant		$F$	96,485	coulombs/(mol)
gas constant	metric	$\bar{R}$	8,314	J/(kmol $\cdot$ K)
gas constant	metric	$\bar{R}$	8.314	kPa $\cdot$ m <sup>3</sup> /(kmol $\cdot$ K)
gas constant	USCS	$\bar{R}$	1,545	ft-lbf/(lb mole $\cdot$ $^{\circ}$ R)
		$\bar{R}$	0.08206	L-atm/(mole $\cdot$ K)
gravitation–Newtonian constant		$G$	$6.673 \times 10^{-11}$	m <sup>3</sup> /(kg $\cdot$ s <sup>2</sup> )
gravitation–Newtonian constant		$G$	$6.673 \times 10^{-11}$	N $\cdot$ m <sup>2</sup> /kg <sup>2</sup>
gravity acceleration (standard)	metric	$g$	9.807	m/s <sup>2</sup>
gravity acceleration (standard)	USCS	$g$	32.174	ft/sec <sup>2</sup>
molar volume (ideal gas), $T = 273.15\text{K}$ , $p = 101.3 \text{ kPa}$		$V_m$	22,414	L/kmol
speed of light in vacuum		$c$	299,792,000	m/s
Stefan-Boltzmann constant		$\sigma$	$5.67 \times 10^{-8}$	W/(m <sup>2</sup> $\cdot$ K <sup>4</sup> )

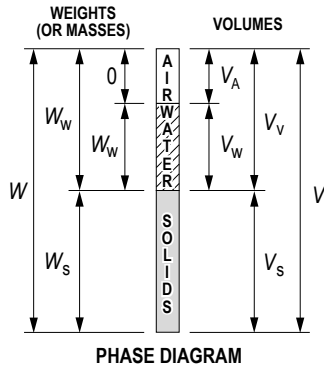
# CONVERSION FACTORS

Multiply	By	To Obtain	Multiply	By	To Obtain
acre	43,560	square feet (ft <sup>2</sup> )	joule (J)	$9.478 \times 10^{-4}$	Btu
ampere-hr (A-hr)	3,600	coulomb (C)	J	0.7376	ft-lbf
ångström (Å)	$1 \times 10^{-10}$	meter (m)	J	1	newton•m (N•m)
atmosphere (atm)	76.0	cm, mercury (Hg)	J/s	1	watt (W)
atm, std	29.92	in., mercury (Hg)			
atm, std	14.70	lb/in <sup>2</sup> abs (psia)	kilogram (kg)	2.205	pound (lbm)
atm, std	33.90	ft, water	kgf	9.8066	newton (N)
atm, std	$1.013 \times 10^5$	pascal (Pa)	kilometer (km)	3,281	feet (ft)
			km/hr	0.621	mph
bar	$1 \times 10^5$	Pa	kilopascal (kPa)	0.145	lb/in <sup>2</sup> (psi)
bar	0.987	atm	kilowatt (kW)	1.341	horsepower (hp)
barrels—oil	42	gallons—oil	kW	3,413	Btu/hr
Btu	1,055	joule (J)	kW	737.6	(ft-lbf )/sec
Btu	$2.928 \times 10^{-4}$	kilowatt-hr (kWh)	kW-hour (kWh)	3,413	Btu
Btu	778	ft-lbf	kWh	1.341	hp-hr
Btu/hr	$3.930 \times 10^{-4}$	horsepower (hp)	kWh	$3.6 \times 10^6$	joule (J)
Btu/hr	0.293	watt (W)	kip (K)	1,000	lbf
Btu/hr	0.216	ft-lbf/sec	K	4,448	newton (N)
calorie (g-cal)	$3.968 \times 10^{-3}$	Btu	liter (L)	61.02	in <sup>3</sup>
cal	$1.560 \times 10^{-6}$	hp-hr	L	0.264	gal (U.S. Liq)
cal	4.186	joule (J)	L	$10^{-3}$	m <sup>3</sup>
cal/sec	4.184	watt (W)	L/second (L/s)	2.119	ft <sup>3</sup> /min (cfm)
centimeter (cm)	$3.281 \times 10^{-2}$	foot (ft)	L/s	15.85	gal (U.S.)/min (gpm)
cm	0.394	inch (in)			
centipoise (cP)	0.001	pascal•sec (Pa•s)	meter (m)	3.281	feet (ft)
centipoise (cP)	1	g/(m•s)	m	1.094	yard
centipoise (cP)	2.419	lbm/hr-ft	m/second (m/s)	196.8	feet/min (ft/min)
centistoke (cSt)	$1 \times 10^{-6}$	m <sup>2</sup> /sec (m <sup>2</sup> /s)	mile (statute)	5,280	feet (ft)
cubic feet/second (cfs)	0.646317	million gallons/day (MGD)	mile (statute)	1.609	kilometer (km)
cubic foot (ft <sup>3</sup> )	7.481	gallon	mile/hour (mph)	88.0	ft/min (fpm)
cubic meters (m <sup>3</sup> )	1,000	liters	mph	1.609	km/h
electronvolt (eV)	$1.602 \times 10^{-19}$	joule (J)	mm of Hg	$1.316 \times 10^{-3}$	atm
			mm of H <sub>2</sub> O	$9.678 \times 10^{-5}$	atm
foot (ft)	30.48	cm	newton (N)	0.225	lbf
ft	0.3048	meter (m)	newton (N)	1	kg•m/s <sup>2</sup>
ft-pound (ft-lbf)	$1.285 \times 10^{-3}$	Btu	N•m	0.7376	ft-lbf
ft-lbf	$3.766 \times 10^{-7}$	kilowatt-hr (kWh)	N•m	1	joule (J)
ft-lbf	0.324	calorie (g-cal)			
ft-lbf	1.356	joule (J)	pascal (Pa)	$9.869 \times 10^{-6}$	atmosphere (atm)
			Pa	1	newton/m <sup>2</sup> (N/m <sup>2</sup> )
ft-lbf/sec	$1.818 \times 10^{-3}$	horsepower (hp)	Pa•sec (Pa•s)	10	poise (P)
			pound (lbm, avdp)	0.454	kilogram (kg)
gallon (U.S. Liq)	3.785	liter (L)	lbf	4.448	N
gallon (U.S. Liq)	0.134	ft <sup>3</sup>	lbf-ft	1.356	N•m
gallons of water	8.3453	pounds of water	lbf/in <sup>2</sup> (psi)	0.068	atm
gamma (γ, Γ)	$1 \times 10^{-9}$	tesla (T)	psi	2.307	ft of H <sub>2</sub> O
gauss	$1 \times 10^{-4}$	T	psi	2.036	in. of Hg
gram (g)	$2.205 \times 10^{-3}$	pound (lbm)	psi	6,895	Pa
hectare	$1 \times 10^4$	square meters (m <sup>2</sup> )	radian	$180/\pi$	degree
hectare	2.47104	acres			
horsepower (hp)	42.4	Btu/min	stokes	$1 \times 10^{-4}$	m <sup>2</sup> /s
hp	745.7	watt (W)			
hp	33,000	(ft-lbf)/min	therm	$1 \times 10^5$	Btu
hp	550	(ft-lbf)/sec	ton (metric)	1,000	kilogram (kg)
hp-hr	2,545	Btu	ton (short)	2,000	pound (lb)
hp-hr	$1.98 \times 10^6$	ft-lbf			
hp-hr	$2.68 \times 10^6$	joule (J)	watt (W)	3.413	Btu/hr
hp-hr	0.746	kWh	W	$1.341 \times 10^{-3}$	horsepower (hp)
			W	1	joule/s (J/s)
inch (in.)	2.540	centimeter (cm)	weber/m <sup>2</sup> (Wb/m <sup>2</sup> )	10,000	gauss
in. of Hg	0.0334	atm			
in. of Hg	13.60	in. of H <sub>2</sub> O			
in. of H <sub>2</sub> O	0.0361	lb/in <sup>2</sup> (psi)			
in. of H <sub>2</sub> O	0.002458	atm			



# CIVIL ENGINEERING

## GEOTECHNICAL Phase Relationships



Volume of voids

$$V_v = V_A + V_w$$

Total unit weight

$$\gamma = W/V$$

Saturated unit weight

$$\gamma_{sat} = (G_s + e) \gamma_w / (1 + e) = \gamma(G_s + e) / (1 + \omega)$$

$$\gamma_w = 62.4 \text{ lb/ft}^3 \text{ or } 9.81 \text{ kN/m}^3$$

Effective (submerged) unit weight

$$\gamma' = \gamma_{sat} - \gamma_w$$

Unit weight of solids

$$\gamma_s = W_s / V_s$$

Dry unit weight

$$\gamma_D = W_s / V$$

Water content (%)

$$\omega = (W_w / W_s) \times 100$$

Specific gravity of soil solids

$$G_s = (W_s / V_s) / \gamma_w$$

Void ratio

$$e = V_v / V_s$$

Porosity

$$n = V_v / V = e / (1 + e)$$

Degree of saturation (%)

$$S = (V_w / V_v) \times 100$$

$$S = \omega G_s / e$$

Relative density

$$D_r = [(e_{max} - e) / (e_{max} - e_{min})] \times 100$$

$$= [(\gamma_{D \text{ field}} - \gamma_{D \text{ min}}) / (\gamma_{D \text{ max}} - \gamma_{D \text{ min}})] [\gamma_{D \text{ max}} / \gamma_{D \text{ field}}] \times 100$$

Relative compaction (%)

$$RC = (\gamma_{D \text{ field}} / \gamma_{D \text{ max}}) \times 100$$

Plasticity index

$$PI = LL - PL$$

LL = liquid limit

PL = Plastic limit

Coefficient of uniformity

$$C_u = D_{60} / D_{10}$$

Coefficient of concavity (or curvature)

$$C_c = (D_{30})^2 / (D_{10} \times D_{60})$$

Hydraulic conductivity (also coefficient of permeability)

From constant head test:  $k = Q / (iAt_e)$

$$i = dh/dL$$

$Q$  = total quantity of water

From falling head test:  $k = 2.303[(aL)/(At_e)] \log_{10}(h_1/h_2)$

$A$  = cross-sectional area of test specimen perpendicular to flow

$a$  = cross-sectional area of reservoir tube

$t_e$  = elapsed time

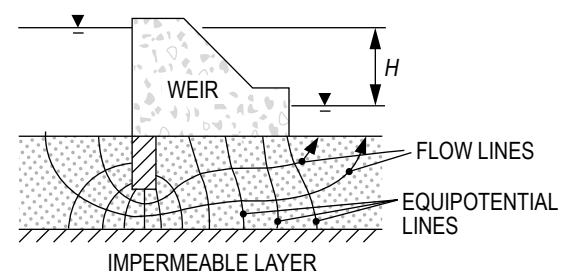
$h_1$  = head at time  $t = 0$

$h_2$  = head at time  $t = t_e$

$L$  = length of soil column

Discharge velocity,  $v = ki$

## Flow Nets



$$Q = kH (N_f / N_d), \text{ where}$$

$Q$  = flow per unit time

$N_f$  = number of flow channels

$N_d$  = number of equipotential drops

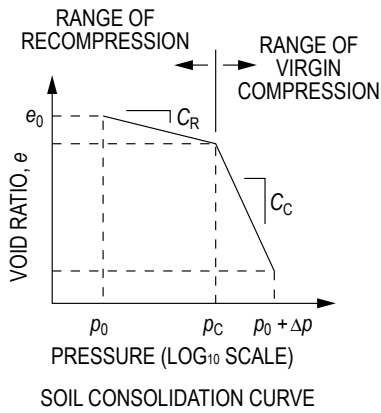
$H$  = total hydraulic head differential

Factor of safety against seepage liquifaction

$$FS_s = i_c / i_e$$

$$i_c = (\gamma_{sat} - \gamma_w) / \gamma_w$$

$i_e$  = seepage exit gradient



$e_0$  = initial void ratio (prior to consolidation)

$\Delta e$  = change in void ratio

$p_0$  = initial effective consolidation stress,  $\sigma'_0$

$p_c$  = past maximum consolidation stress,  $\sigma'_c$

$\Delta p$  = induced change in consolidation stress at center of consolidating stratum

$$\Delta p = I q_s$$

$I$  = Stress influence value at center of consolidating stratum

$q_s$  = applied surface stress causing consolidation

$$\text{If } (p_0 \text{ and } p_0 + \Delta p) < p_c, \text{ then } \Delta H = \frac{H_0}{1 + e_0} \left[ C_R \log \frac{p_0 + \Delta p}{p_0} \right]$$

$$\text{If } (p_0 \text{ and } p_0 + \Delta p) > p_c, \text{ then } \Delta H = \frac{H_0}{1 + e_0} \left[ C_C \log \frac{p_0 + \Delta p}{p_0} \right]$$

$$\text{If } p_0 < p_c < (p_0 + \Delta p), \text{ then } \Delta H = \frac{H_0}{1 + e_0} \left[ C_R \log \frac{p_c}{p_0} + C_C \log \frac{p_0 + \Delta p}{p_c} \right]$$

where:  $\Delta H$  = change in thickness of soil layer

Compression index

In virgin compression range:  $C_C = \Delta e / \Delta \log p$

By correlation to liquid limit:  $C_C = 0.009 (LL - 10)$

Recompression index

In recompression range:  $C_R = \Delta e / \Delta \log p$

By correlation to compression index,  $C_C$ :  $C_R = C_C / 6$

Ultimate consolidation settlement in soil layer

$$S_{ULT} = \epsilon_v H_s$$

$H_s$  = thickness of soil layer

$$\epsilon_v = \Delta e_{TOT} / (1 + e_0)$$

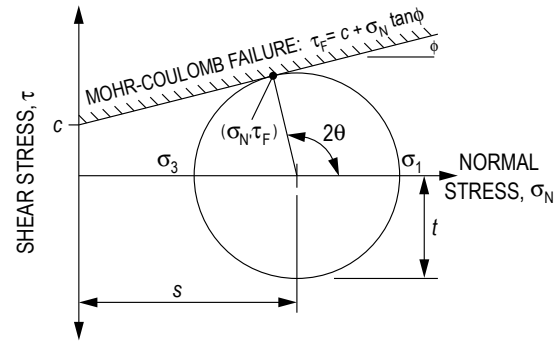
$\Delta e_{TOT}$  = total change in void ratio due to recompression and virgin compression

Approximate settlement (at time  $t = t_c$ )

$$S_T = U_{AV} S_{ULT}$$

$U_{AV}$  = average degree of consolidation

$t_c$  = elapsed time since application of consolidation load



$s$  = mean normal stress

$t$  = maximum shear stress

$\sigma_1$  = major principal stress

$\sigma_3$  = minor principal stress

$\theta$  = orientation angle between plane of existing normal stress and plane of major principal stress

Total normal stress

$$\sigma_N = P/A$$

$P$  = normal force

$A$  = cross-sectional area over which force acts

Effective stress

$$\sigma' = \sigma - u$$

$$u = h_u \gamma_w$$

$h_u$  = uplift or pressure head

Shear stress

$$\tau = T/A$$

$T$  = shearing force

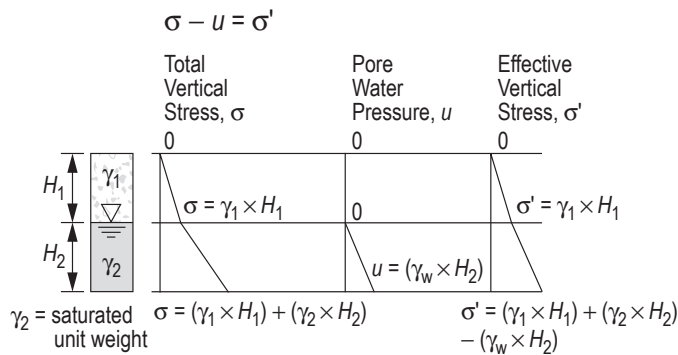
Shear stress at failure

$$\tau_F = c + \sigma_N \tan \phi$$

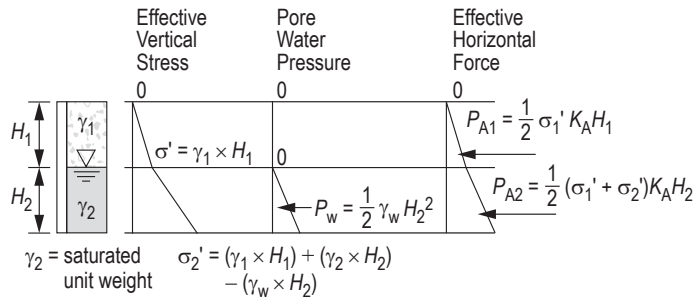
$c$  = cohesion

$\phi$  = angle of internal friction

## Vertical Stress Profiles



## Horizontal Stress Profiles and Forces



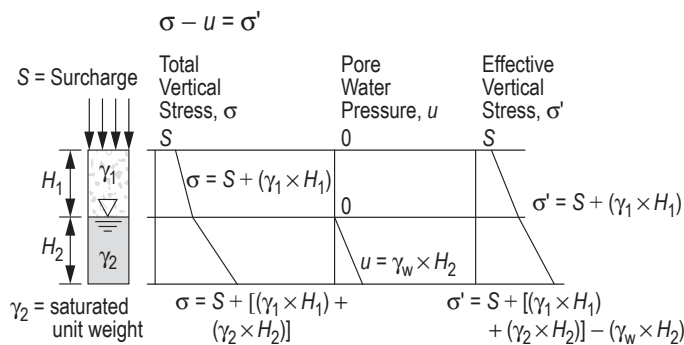
Active forces on retaining wall per unit wall length (as shown):

$K_A$  = Rankine active earth pressure coefficient (smooth wall,  $c = 0$ , level backfill) =  $\tan^2 (45^\circ - \phi/2)$

Passive forces on retaining wall per unit wall length (similar to the active forces shown):

$K_p$  = Rankine passive earth pressure coefficient (smooth wall,  $c = 0$ , level backfill) =  $\tan^2 (45^\circ + \phi/2)$

## Vertical Stress Profiles with Surcharge



## Ultimate Bearing Capacity

$$q_{ULT} = cN_c + \gamma' D_f N_q + \frac{1}{2} \gamma' B N_\gamma$$

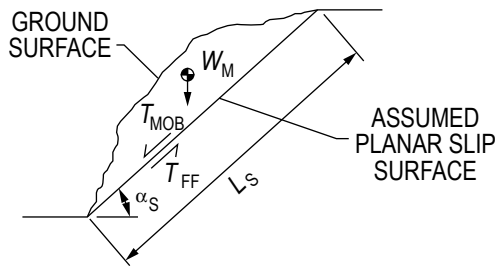
$N_c$  = bearing capacity factor for cohesion

$N_q$  = bearing capacity factor for depth

$N_\gamma$  = bearing capacity factor for unit weight

$D_f$  = depth of footing below ground surface

$B$  = width of strip footing



### SLOPE FAILURE ALONG PLANAR SURFACE

- FS = factor of safety against slope instability  
 =  $T_{FF}/T_{MOB}$   
 $T_{FF}$  = available shearing resistance along slip surface  
 =  $cL_S + W_M \cos\alpha_S \tan\phi$   
 $T_{MOB}$  = mobilized shear force along slip surface  
 =  $W_M \sin\alpha_S$   
 $L_S$  = length of assumed planar slip surface  
 $W_M$  = weight of soil above slip surface  
 $\alpha_S$  = angle of assumed slip surface with respect to horizontal

### AASHTO Soil Classification

GENERAL CLASSIFICATION	GRANULAR MATERIALS ( 35% OR LESS PASSING 0.075 SIEVE )							SILT-CLAY MATERIALS ( MORE THAN 35% PASSING 0.075 SIEVE )			
GROUP CLASSIFICATION	A-1		A-3	A-2				A-4	A-5	A-6	A-7-5 A-7-6
	A-1-a	A-1-b		A-2-4	A-2-5	A-2-6	A-2-7				
SIEVE ANALYSIS, PERCENT PASSING: 2.00 mm (No. 10) 0.425 mm (No. 40) 0.075 mm (No. 200)	≤ 50 ≤ 30 ≤ 15	— ≤ 50 ≤ 25	— ≥ 51 ≤ 10	— — ≤ 35	— — ≤ 35	— — ≤ 35	— — ≤ 35	— — ≥ 36	— — ≥ 36	— — ≥ 36	— — ≥ 36
CHARACTERISTICS OF FRACTION PASSING 0.425 SIEVE (No. 40): LIQUID LIMIT PLASTICITY INDEX *	— 6 max		— NP	≤ 40 ≤ 10	≥ 41 ≤ 10	≤ 40 ≥ 11	≥ 41 ≥ 11	≤ 40 ≤ 10	≥ 41 ≤ 10	≤ 40 ≥ 11	≥ 41 ≥ 11
USUAL TYPES OF CONSTITUENT MATERIALS	STONE FRAGM'TS, GRAVEL, SAND		FINE SAND	SILTY OR CLAYEY GRAVEL AND SAND				SILTY SOILS		CLAYEY SOILS	
GENERAL RATING AS A SUBGRADE	EXCELLENT TO GOOD							FAIR TO POOR			

\*Plasticity index of A-7-5 subgroup is equal to or less than  $LL - 30$ . Plasticity index of A-7-6 subgroup is greater than  $LL - 30$ .  
 NP = Non-plastic (use "0"). Symbol "—" means that the particular sieve analysis is not considered for that classification.

If the soil classification is A4-A7, then calculate the group index (GI) as shown below and report with classification. The higher the GI, the less suitable the soil. Example: A-6 with GI = 15 is less suitable than A-6 with GI = 10.

$$GI = (F - 35) [ 0.2 + 0.005 (LL - 40) ] + 0.01 (F - 15) (PI - 10)$$

- where: F = Percent passing No. 200 sieve, expressed as a whole number. This percentage is based only on the material passing the No. 200 sieve.  
 LL = Liquid limit  
 PI = Plasticity index

If the computed value of  $GI < 0$ , then use  $GI = 0$ .

# ASTM D2487-11 Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification System)

Criteria for Assigning Group Symbols and Group Names Using Laboratory Tests <sup>A</sup>				Soil Classification		
				Group Symbol	Group Name <sup>B</sup>	
COARSE-GRAINED SOILS	Gravels (more than 50% of coarse fraction retained on No. 4 sieve)	Clean Gravels (Less than 5% fines <sup>C</sup> )	$Cu \geq 4$ and $1 \leq Cc \leq 3^D$	GW	Well-graded gravel <sup>E</sup>	
			$Cu < 4$ and/or $[Cc < 1 \text{ or } Cc > 3]^D$	GP	Poorly graded gravel <sup>E</sup>	
		Gravels with Fines (More than 12% fines <sup>C</sup> )	Fines classify as ML or MH	GM	Silty gravel <sup>E, F, G</sup>	
			Fines classify as CL or CH	GC	Clayey gravel <sup>E, F, G</sup>	
	More than 50% retained on No. 200 sieve	Sands (50% or more of coarse fraction passes No. 4 sieve)	Clean Sands (Less than 5% fines <sup>H</sup> )	$Cu \geq 6$ and $1 \leq Cc \leq 3^D$	SW	Well-graded sand <sup>I</sup>
				$Cu < 6$ and/or $[Cc < 1 \text{ or } Cc > 3]^D$	SP	Poorly graded sand <sup>I</sup>
Sands with Fines (More than 12% fines <sup>H</sup> )	Fines classify as ML or MH		SM	Silty sand <sup>F, G, I</sup>		
	Fines classify as CL or CH		SC	Clayey sand <sup>F, G, I</sup>		
FINE-GRAINED SOILS	Silts and Clays	inorganic	$PI > 7$ and plots on or above "A" line <sup>J</sup>	CL	Lean clay <sup>K, L, M</sup>	
			$PI < 4$ or plots below "A" line <sup>J</sup>	ML	Silt <sup>K, L, M</sup>	
	Liquid limit less than 50	organic	Liquid limit - oven dried/Liquid&#10	OL	Organic clay <sup>K, L, M, N</sup>	
			$< 0.75$		Organic silt <sup>K, L, M, O</sup>	
	50% or more passes the No. 200 sieve	Silts and Clays	inorganic	PI plots on or above "A" line	CH	Fat clay <sup>K, L, M</sup>
				PI plots below "A" line	MH	Elastic silt <sup>K, L, M</sup>
		Liquid limit 50 or more	organic	Liquid limit - oven dried/Liquid&#10	OH	Organic clay <sup>K, L, M, P</sup>
				$< 0.75$		Organic silt <sup>K, L, M, Q</sup>
HIGHLY ORGANIC SOILS		Primarily organic matter, dark in color, and organic odor		PT	Peat	

<sup>A</sup>Based on the material passing the 3-in. (75-mm) sieve.

<sup>B</sup>If field sample contained cobbles or boulders, or both, add "with cobbles or boulders, or both" to group name.

<sup>C</sup>Gravels with 5 to 12% fines require dual symbols:

GW-GM well-graded gravel with silt  
GW-GC well-graded gravel with clay  
GP-GM poorly graded gravel with silt  
GP-GC poorly graded gravel with clay

$$^D Cu = D_{60}/D_{10} \quad Cc = \frac{(D_{30})^2}{D_{10} \times D_{60}}$$

<sup>E</sup>If soil contains  $\geq 15\%$  sand, add "with sand" to group name.

<sup>F</sup>If fines classify as CL-ML, use dual symbol GC-GM, or SC-SM.

<sup>G</sup>If fines are organic, add "with organic fines" to group name.

<sup>H</sup>Sands with 5 to 12% fines require dual symbols:

SW-SM well-graded sand with silt  
SW-SC well-graded sand with clay  
SP-SM poorly graded sand with silt  
SP-SC poorly graded sand with clay

<sup>I</sup>If soil contains  $\geq 15\%$  gravel, add "with gravel" to group name.

<sup>J</sup>If Atterberg limits plot in hatched area, soil is a CL-ML, silty clay.

<sup>K</sup>If soil contains 15 to 30% plus No. 200, add "with sand" or "with gravel", whichever is predominant.

<sup>L</sup>If soil contains  $\geq 30\%$  plus No. 200, predominantly sand, add "sandy" to group name.

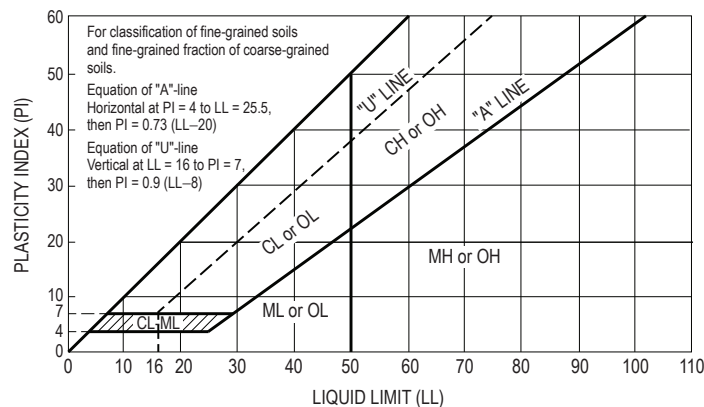
<sup>M</sup>If soil contains  $\geq 30\%$  plus No. 200, predominantly gravel, add "gravelly" to group name.

<sup>N</sup> $PI \geq 4$  and plots on or above "A" line.

<sup>O</sup> $PI < 4$  or plots below "A" line.

<sup>P</sup>PI plots on or above "A" line.

<sup>Q</sup>PI plots below "A" line.

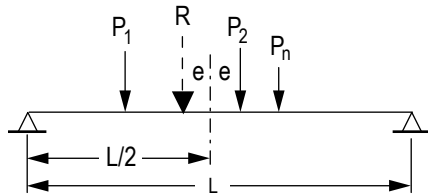


## STRUCTURAL ANALYSIS

### Influence Lines for Beams and Trusses

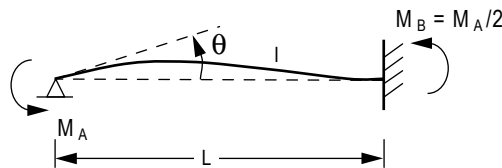
An influence line shows the variation of an effect (reaction, shear and moment in beams, bar force in a truss) caused by moving a unit load across the structure. An influence line is used to determine the position of a moveable set of loads that causes the maximum value of the effect.

### Moving Concentrated Load Sets



The **absolute maximum moment** produced in a beam by a set of "n" moving loads occurs when the resultant "R" of the load set and an adjacent load are equal distance from the centerline of the beam. In general, two possible load set positions must be considered, one for each adjacent load.

### Beam Stiffness and Moment Carryover



$$\theta = \frac{M L}{4 E I} \Rightarrow M = \left( \frac{4 E I}{L} \right) \theta = k_{AB} \theta$$

$k_{AB}$  = stiffness

$M_B = M_A/2$  = carryover

### Truss Deflection by Unit Load Method

The displacement of a truss joint caused by external effects (truss loads, member temperature change, member misfit) is found by applying a unit load at the point that corresponds to the desired displacement.

$$\Delta_{\text{joint}} = \sum_{i=1}^{\text{members}} f_i (\Delta L)_i$$

where:  $\Delta_{\text{joint}}$  = joint displacement at point of application of unit load (+ in direction of unit load)

$f_i$  = force in member "i" caused by unit load (+ tension)

$(\Delta L)_i$  = change in length caused by external effect (+ for increase in member length):

$$= \left( \frac{F L}{A E} \right)_i \text{ for bar force } F \text{ caused by external load}$$

$$= \alpha L_i (\Delta T)_i \text{ for temperature change in member } (\alpha = \text{coefficient of thermal expansion})$$

= member misfit

$L, A$  = member length and cross-sectional area

$E$  = member elastic modulus

### Frame Deflection by Unit Load Method

The displacement of any point on a frame caused by external loads is found by applying a unit load at that point that corresponds to the desired displacement:

$$\Delta = \sum_{i=1}^{\text{members}} \int_{x=0}^{x=L_i} \frac{m_i M_i}{E I_i} dx$$

where:  $\Delta$  = displacement at point of application of unit load (+ in direction of unit load)

$m_i$  = moment equation in member "i" caused by the unit load

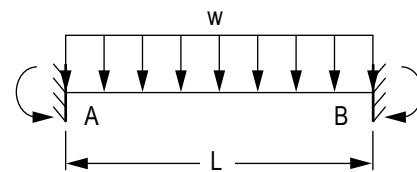
$M_i$  = moment equation in member "i" caused by loads applied to frame

$L_i$  = length of member "i"

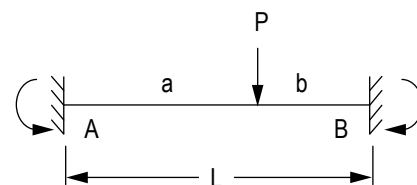
$I_i$  = moment of inertia of member "i"

If either the real loads or the unit load cause no moment in a member, that member can be omitted from the summation.

### Member Fixed-End Moments (Magnitudes)



$$FEM_{AB} = FEM_{BA} = \frac{w L^2}{12}$$



$$FEM_{AB} = \frac{P a b^2}{L^2} \quad FEM_{BA} = \frac{P a^2 b}{L^2}$$

## STABILITY, DETERMINANCY, AND CLASSIFICATION OF STRUCTURES

$m$  = number of members

$r$  = number of independent reaction components

$j$  = number of joints

$c$  = number of condition equations based on known internal moments or forces, such as internal moment of zero at a hinge

### Plane Truss

<u>Static Analysis</u>	<u>Classification</u>
------------------------	-----------------------

$m + r < 2j$	Unstable
--------------	----------

$m + r = 2j$	Stable and statically determinate
--------------	-----------------------------------

$m + r > 2j$	Stable and statically indeterminate
--------------	-------------------------------------

### Plane Frame

<u>Static Analysis</u>	<u>Classification</u>
------------------------	-----------------------

$3m + r < 3j + c$	Unstable
-------------------	----------

$3m + r = 3j + c$	Stable and statically determinate
-------------------	-----------------------------------

$3m + r > 3j + c$	Stable and statically indeterminate
-------------------	-------------------------------------

Stability also requires an appropriate arrangement of members and reaction components.

## STRUCTURAL DESIGN

### Live Load Reduction

The effect on a building member of nominal occupancy live loads may often be reduced based on the loaded floor area supported by the member. A typical model used for computing reduced live load (as found in ASCE 7 and many building codes) is:

$$L_{\text{reduced}} = L_{\text{nominal}} \left( 0.25 + \frac{15}{\sqrt{K_{LL} A_T}} \right) \geq 0.4 L_{\text{nominal}}$$

where:  $L_{\text{nominal}}$  = nominal live load given in ASCE 7 or a building code

$A_T$  = the cumulative floor tributary area supported by the member

$K_{LL} A_T$  = area of influence supported by the member

$K_{LL}$  = ratio of area of influence to the tributary area supported by the member:

$K_{LL} = 4$  (typical columns)

$K_{LL} = 2$  (typical beams and girders)

## Load Combinations using Strength Design (LRFD, USD)

Nominal loads used in following combinations:

$D$  = dead loads

$E$  = earthquake loads

$L$  = live loads (floor)

$L_r$  = live loads (roof)

$R$  = rain load

$S$  = snow load

$W$  = wind load

Load factors  $\lambda$ :  $\lambda_D$  (dead load),  $\lambda_L$  (live load), etc.

Basic combinations  $L_r/S/R$  = largest of  $L_r$ ,  $S$ ,  $R$

$L$  or  $0.8W$  = larger of  $L$ ,  $0.8W$

$1.4D$

$1.2D + 1.6L + 0.5(L_r/S/R)$

$1.2D + 1.6(L_r/S/R) + (L \text{ or } 0.8W)$

$1.2D + 1.6W + L + 0.5(L_r/S/R)$

$1.2D + 1.0E + L + 0.2S$

$0.9D + 1.6W$

$0.9D + 1.0E$

## DESIGN OF REINFORCED CONCRETE COMPONENTS (ACI 318-11)

U.S. Customary units

### Definitions

$a$  = depth of equivalent rectangular stress block, in.

$A_g$  = gross area of column, in<sup>2</sup>

$A_s$  = area of tension reinforcement, in<sup>2</sup>

$A_{st}$  = total area of longitudinal reinforcement, in<sup>2</sup>

$A_v$  = area of shear reinforcement within a distance  $s$ , in.

$b$  = width of compression face of member, in.

$\beta_1$  = ratio of depth of rectangular stress block,  $a$ , to depth to neutral axis,  $c$

$$= 0.85 \geq 0.85 - 0.05 \left( \frac{f'_c - 4,000}{1,000} \right) \geq 0.65$$

$c$  = distance from extreme compression fiber to neutral axis, in.

$d$  = distance from extreme compression fiber to centroid of nonprestressed tension reinforcement, in.

$d_t$  = distance from extreme compression fiber to extreme tension steel, in.

$E_c$  = modulus of elasticity =  $33w_c^{1.5} \sqrt{f'_c}$ , psi

$\epsilon_t$  = net tensile strain in extreme tension steel at nominal strength

$f'_c$  = compressive strength of concrete, psi

$f_y$  = yield strength of steel reinforcement, psi

$M_n$  = nominal moment strength at section, in.-lb

$\phi M_n$  = design moment strength at section, in.-lb

$M_u$  = factored moment at section, in.-lb

$P_n$  = nominal axial load strength at given eccentricity, lb

$\phi P_n$  = design axial load strength at given eccentricity, lb

$P_u$  = factored axial force at section, lb

$\rho_g$  = ratio of total reinforcement area to cross-sectional area of column =  $A_{st}/A_g$

$s$  = spacing of shear ties measured along longitudinal axis of member, in.

$V_c$  = nominal shear strength provided by concrete, lb

$V_n$  = nominal shear strength at section, lb

$\phi V_n$  = design shear strength at section, lb

$V_s$  = nominal shear strength provided by reinforcement, lb

$V_u$  = factored shear force at section, lb

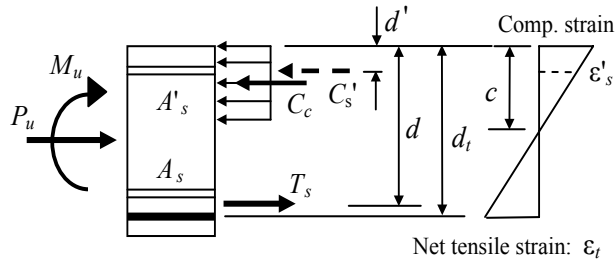
### ASTM STANDARD REINFORCING BARS

BAR SIZE	DIAMETER, IN.	AREA, IN <sup>2</sup>	WEIGHT, LB/FT
#3	0.375	0.11	0.376
#4	0.500	0.20	0.668
#5	0.625	0.31	1.043
#6	0.750	0.44	1.502
#7	0.875	0.60	2.044
#8	1.000	0.79	2.670
#9	1.128	1.00	3.400
#10	1.270	1.27	4.303
#11	1.410	1.56	5.313
#14	1.693	2.25	7.650
#18	2.257	4.00	13.60

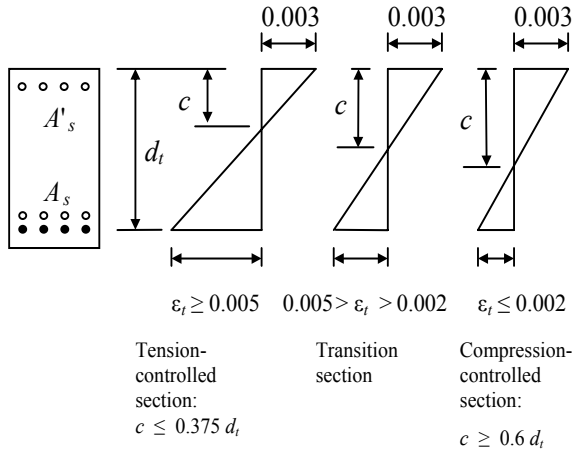


## UNIFIED DESIGN PROVISIONS

### Internal Forces and Strains



### Strain Conditions



## RESISTANCE FACTORS, $\phi$

Tension-controlled sections ( $\varepsilon_t \geq 0.005$ ):  $\phi = 0.9$

Compression-controlled sections ( $\varepsilon_t \leq 0.002$ ):

Members with tied reinforcement  $\phi = 0.65$

Transition sections ( $0.002 < \varepsilon_t < 0.005$ ):

Members with tied reinforcement  $\phi = 0.48 + 83\varepsilon_t$

Shear and torsion  $\phi = 0.75$

Bearing on concrete  $\phi = 0.65$

## BEAMS—FLEXURE

$$\phi M_n \geq M_u$$

### For All Beams

Net tensile strain:  $a = \beta_1 c$

$$\varepsilon_t = \frac{0.003(d_t - c)}{c} = \frac{0.003(\beta_1 d_t - a)}{a}$$

Design moment strength:  $\phi M_n$

where:  $\phi = 0.9$  [ $\varepsilon_t \geq 0.005$ ]

$$\phi = 0.48 + 83\varepsilon_t [0.004 \leq \varepsilon_t < 0.005]$$

### Singly-Reinforced Beams

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = 0.85 f'_c a b \left( d - \frac{a}{2} \right) = A_s f_y \left( d - \frac{a}{2} \right)$$

## BEAMS—SHEAR

$$\phi V_n \geq V_u$$

Nominal shear strength:

$$V_n = V_c + V_s$$

$$V_c = 2 b_w d \sqrt{f'_c}$$

$$V_s = \frac{A_v f_y d}{s} \text{ (may not exceed } 8 b_w d \sqrt{f'_c} \text{)}$$

Required and maximum-permitted stirrup spacing,  $s$

$$V_u \leq \frac{\phi V_c}{2}: \text{ No stirrups required}$$

$$V_u > \frac{\phi V_c}{2}: \text{ Use the following table (} A_v \text{ given)}$$

	$\frac{\phi V_c}{2} < V_u \leq \phi V_c$	$V_u > \phi V_c$
Required spacing	<p>Smaller of:</p> $s = \frac{A_v f_y}{50 b_w}$ $s = \frac{A_v f_y}{0.75 b_w \sqrt{f'_c}}$	$V_s = \frac{V_u}{\phi} - V_c$ $s = \frac{A_v f_y d}{V_s}$
Maximum permitted spacing	<p>Smaller of:</p> $s = \frac{d}{2}$ <p>OR</p> $s = 24"$	$V_s \leq 4 b_w d \sqrt{f'_c}$ <p>Smaller of:</p> $s = \frac{d}{2} \quad \text{OR}$ $s = 24"$
		$V_s > 4 b_w d \sqrt{f'_c}$ <p>Smaller of:</p> $s = \frac{d}{4}$ $s = 12"$

## SHORT COLUMNS

### Limits for Main Reinforcements

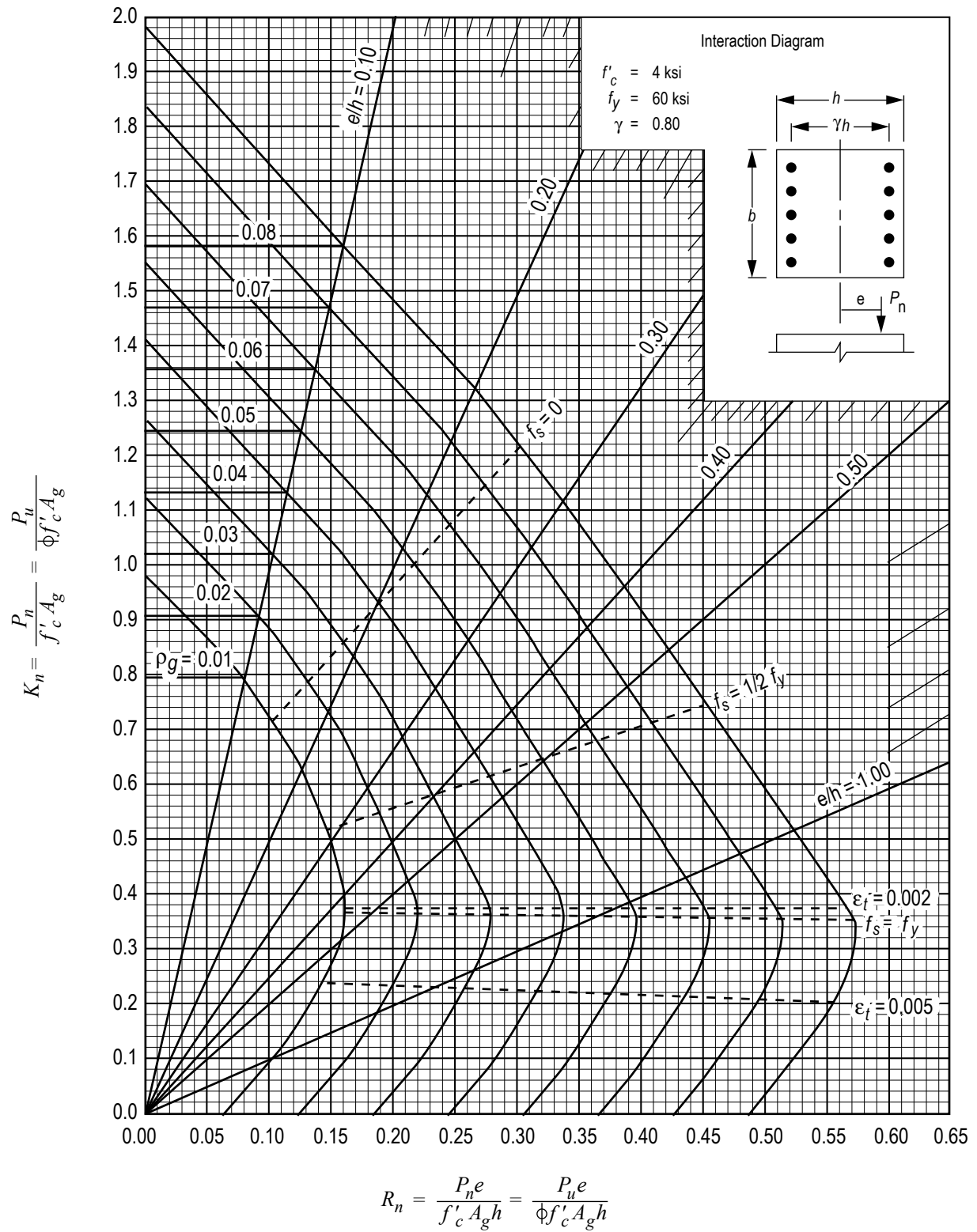
$$\rho_g = \frac{A_{st}}{A_g}$$

$$0.01 \leq \rho_g \leq 0.08$$

### Design Column Strength, Tied Columns

$$\phi = 0.65$$

$$\phi P_n = 0.80 \phi [0.85 f'_c (A_g - A_{st}) + A_{st} f_y]$$



### GRAPH A.11

Column strength interaction diagram for rectangular section with bars on end faces and  $\gamma = 0.80$  (for instructional use only).

Nilson, Arthur H., David Darwin, and Charles W. Dolan, *Design of Concrete Structures*, 13th ed., McGraw-Hill, New York, 2004.

## DESIGN OF STEEL COMPONENTS

(ANSI/AISC 360-10)

LRFD,  $E = 29,000$  ksi

### BEAMS

For doubly symmetric compact I-shaped members bent about their major axis, the *design flexural strength*  $\phi_b M_n$  is determined with  $\phi_b = 0.90$  as follows:

#### Yielding

$$M_n = M_p = F_y Z_x$$

where

$F_y$  = specified minimum yield stress

$Z_x$  = plastic section modulus about the x-axis

#### Lateral-Torsional Buckling

Based on bracing where  $L_b$  is the length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section with respect to the length limits  $L_p$  and  $L_r$ :

When  $L_b \leq L_p$ , the limit state of lateral-torsional buckling does not apply.

When  $L_p < L_b \leq L_r$

$$M_n = C_b \left[ M_p - (M_p - 0.7 F_y S_x) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p$$

where

$$C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3 M_A + 4 M_B + 3 M_C}$$

$M_{\max}$  = absolute value of maximum moment in the unbraced segment

$M_A$  = absolute value of maximum moment at quarter point of the unbraced segment

$M_B$  = absolute value of maximum moment at centerline of the unbraced segment

$M_C$  = absolute value of maximum moment at three-quarter of the unbraced segment

#### Shear

The *design shear strength*  $\phi_v V_n$  is determined with  $\phi_v = 1.00$  for webs of rolled I-shaped members and is determined as follows:

$$V_n = 0.6 F_y (d t_w)$$

## COLUMNS

The *design compressive strength*  $\phi_c P_n$  is determined with  $\phi_c = 0.90$  for flexural buckling of members without slender elements and is determined as follows:

$$P_n = F_{cr} A_g$$

where the critical stress  $F_{cr}$  is determined as follows:

$$(a) \text{ When } \frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}}, F_{cr} = \left[ 0.658 \frac{F_y}{F_c} \right] F_y$$

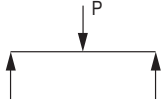
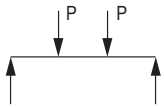
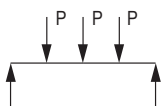
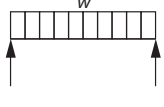
$$(b) \text{ When } \frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}}, F_{cr} = 0.877 F_e$$

where

$KL/r$  is the effective slenderness ratio based on the column effective length ( $KL$ ) and radius of gyration ( $r$ )

$KL$  is determined from AISC Table C-A-7.1 or AISC Figures C-A-7.1 and C-A-7.2 on p. 158.

$F_e$  is the elastic buckling stress =  $\pi^2 E / (KL/r)^2$

VALUES OF $C_b$ FOR SIMPLY SUPPORTED BEAMS		
LOAD	LATERAL BRACING ALONG SPAN	$C_b$
	NONE LOAD AT MIDPOINT	1.32
	AT LOAD POINT	1.67
	NONE LOADS AT THIRD POINTS	1.14
	AT LOAD POINTS LOADS SYMMETRICALLY PLACED	1.67
	NONE LOADS AT QUARTER POINTS	1.14
	AT LOAD POINTS LOADS AT QUARTER POINTS	1.67
	NONE	1.14
	AT MIDPOINT	1.30
	AT THIRD POINTS	1.45
	AT QUARTER POINTS	1.52
	AT FIFTH POINTS	1.56

NOTE: LATERAL BRACING MUST ALWAYS BE PROVIDED AT POINTS OF SUPPORT PER AISC SPECIFICATION CHAPTER F.

Adapted from *Steel Construction Manual*, 14th ed., AISC, 2011.

## TENSION MEMBERS

### Flat bars or angles, bolted or welded

#### Definitions

Bolt diameter:  $d_b$

Nominal hole diameter:  $d_h = d_b + 1/16"$

Gross width of member:  $b_g$

Member thickness:  $t$

Connection eccentricity:  $\bar{x}$

Gross area:  $A_g = b_g t$  (use tabulated areas for angles)

Net area (parallel holes):  $A_n = \left[ b_g - \sum \left( d_h + \frac{1}{16} \right) \right] t$

Net area (staggered holes):

$$A_n = \left[ b_g - \sum \left( d_h + \frac{1}{16} \right) + \sum \frac{s^2}{4g} \right] t$$

$s$  = longitudinal spacing of consecutive holes

$g$  = transverse spacing between lines of holes

Effective area (bolted members):

$$A_e = UA_n \quad \begin{cases} \text{Flat bars: } U = 1.0 \\ \text{Angles: } U = 1 - \bar{x}/L \end{cases}$$

Effective area (welded members):

$$A_e = UA_n \quad \begin{cases} \text{Flat bars or angles with transverse} \\ \text{welds: } U = 1.0 \\ \text{Flat bars of width "w", longitudinal} \\ \text{welds of length "L" only:} \\ \quad U = 1.0 \quad (L \geq 2w) \\ \quad U = 0.87 \quad (2w > L \geq 1.5w) \\ \quad U = 0.75 \quad (1.5w > L > w) \\ \text{Angles with longitudinal welds} \\ \text{only} \\ \quad U = 1 - \bar{x}/L \end{cases}$$

### Limit States and Available Strengths

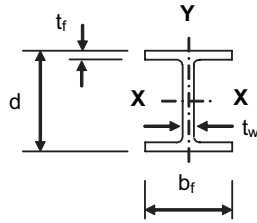
Yielding:  $\phi_y = 0.90$   
 $\phi T_n = \phi_y F_y A_g$

Fracture:  $\phi_f = 0.75$   
 $\phi T_n = \phi_f F_u A_e$

Block shear:  $\phi = 0.75$   
 $U_{bs} = 1.0$  (flat bars and angles)  
 $A_{gv}$  = gross area for shear  
 $A_{nv}$  = net area for shear  
 $A_{nt}$  = net area for tension

$$\phi T_n = \begin{cases} 0.75 F_u [0.6 A_{nv} + U_{bs} A_{nt}] \\ 0.75 [0.6 F_y A_{gv} + U_{bs} F_u A_{nt}] \end{cases}$$

smaller



**Table 1-1: W Shapes Dimensions and Properties**

Shape	Area	Depth	Web	Flange		Axis X-X				Axis Y-Y	
	A	d	t <sub>w</sub>	b <sub>f</sub>	t <sub>f</sub>	I	S	r	Z	I	r
	In. <sup>2</sup>	In.	In.	In.	In.	In. <sup>4</sup>	In. <sup>3</sup>	In.	In. <sup>3</sup>	In. <sup>4</sup>	In.
W24X68	20.1	23.7	0.415	8.97	0.585	1830	154	9.55	177	70.4	1.87
W24X62	18.2	23.7	0.430	7.04	0.590	1550	131	9.23	153	34.5	1.38
W24X55	16.3	23.6	0.395	7.01	0.505	1350	114	9.11	134	29.1	1.34
W21X73	21.5	21.2	0.455	8.30	0.740	1600	151	8.64	172	70.6	1.81
W21X68	20.0	21.1	0.430	8.27	0.685	1480	140	8.60	160	64.7	1.80
W21X62	18.3	21.0	0.400	8.24	0.615	1330	127	8.54	144	57.5	1.77
W21X55	16.2	20.8	0.375	8.22	0.522	1140	110	8.40	126	48.4	1.73
W21X57	16.7	21.1	0.405	6.56	0.650	1170	111	8.36	129	30.6	1.35
W21X50	14.7	20.8	0.380	6.53	0.535	984	94.5	8.18	110	24.9	1.30
W21X48	14.1	20.6	0.350	8.14	0.430	959	93.0	8.24	107	38.7	1.66
W21X44	13.0	20.7	0.350	6.50	0.450	843	81.6	8.06	95.4	20.7	1.26
W18X71	20.8	18.5	0.495	7.64	0.810	1170	127	7.50	146	60.3	1.70
W18X65	19.1	18.4	0.450	7.59	0.750	1070	117	7.49	133	54.8	1.69
W18X60	17.6	18.2	0.415	7.56	0.695	984	108	7.47	123	50.1	1.68
W18X55	16.2	18.1	0.390	7.53	0.630	890	98.3	7.41	112	44.9	1.67
W18X50	14.7	18.0	0.355	7.50	0.570	800	88.9	7.38	101	40.1	1.65
W18X46	13.5	18.1	0.360	6.06	0.605	712	78.8	7.25	90.7	22.5	1.29
W18X40	11.8	17.9	0.315	6.02	0.525	612	68.4	7.21	78.4	19.1	1.27
W16X67	19.7	16.3	0.395	10.2	0.67	954	117	6.96	130	119	2.46
W16X57	16.8	16.4	0.430	7.12	0.715	758	92.2	6.72	105	43.1	1.60
W16X50	14.7	16.3	0.380	7.07	0.630	659	81.0	6.68	92.0	37.2	1.59
W16X45	13.3	16.1	0.345	7.04	0.565	586	72.7	6.65	82.3	32.8	1.57
W16X40	11.8	16.0	0.305	7.00	0.505	518	64.7	6.63	73.0	28.9	1.57
W16X36	10.6	15.9	0.295	6.99	0.430	448	56.5	6.51	64.0	24.5	1.52
W14X74	21.8	14.2	0.450	10.1	0.785	795	112	6.04	126	134	2.48
W14X68	20.0	14.0	0.415	10.0	0.720	722	103	6.01	115	121	2.46
W14X61	17.9	13.9	0.375	9.99	0.645	640	92.1	5.98	102	107	2.45
W14X53	15.6	13.9	0.370	8.06	0.660	541	77.8	5.89	87.1	57.7	1.92
W14X48	14.1	13.8	0.340	8.03	0.595	484	70.2	5.85	78.4	51.4	1.91
W12X79	23.2	12.4	0.470	12.1	0.735	662	107	5.34	119	216	3.05
W12X72	21.1	12.3	0.430	12.0	0.670	597	97.4	5.31	108	195	3.04
W12X65	19.1	12.1	0.390	12.0	0.605	533	87.9	5.28	96.8	174	3.02
W12X58	17.0	12.2	0.360	10.0	0.640	475	78.0	5.28	86.4	107	2.51
W12X53	15.6	12.1	0.345	9.99	0.575	425	70.6	5.23	77.9	95.8	2.48
W12X50	14.6	12.2	0.370	8.08	0.640	391	64.2	5.18	71.9	56.3	1.96
W12X45	13.1	12.1	0.335	8.05	0.575	348	57.7	5.15	64.2	50.0	1.95
W12X40	11.7	11.9	0.295	8.01	0.515	307	51.5	5.13	57.0	44.1	1.94
W10x60	17.6	10.2	0.420	10.1	0.680	341	66.7	4.39	74.6	116	2.57
W10x54	15.8	10.1	0.370	10.0	0.615	303	60.0	4.37	66.6	103	2.56
W10x49	14.4	10.0	0.340	10.0	0.560	272	54.6	4.35	60.4	93.4	2.54
W10x45	13.3	10.1	0.350	8.02	0.620	248	49.1	4.32	54.9	53.4	2.01
W10x39	11.5	9.92	0.315	7.99	0.530	209	42.1	4.27	46.8	45.0	1.98

Adapted from *Steel Construction Manual*, 14th ed., AISC, 2011.

<div> <div><b>Z<sub>x</sub></b></div> <div>AISC Table 3-2</div> <div><b>W Shapes – Selection by Z<sub>x</sub></b></div> <div> F<sub>y</sub> = 50 ksi  φ<sub>b</sub> = 0.90  φ<sub>v</sub> = 1.00 </div> </div>								
Shape	Z <sub>x</sub> in. <sup>3</sup>	φ <sub>b</sub> M <sub>px</sub> kip-ft	φ <sub>b</sub> M <sub>rx</sub> kip-ft	φ <sub>b</sub> BF kips	L <sub>p</sub> ft.	L <sub>r</sub> ft.	I <sub>x</sub> in. <sup>4</sup>	φ <sub>v</sub> V <sub>nx</sub> kips
<b>W24 x 55</b>	<b>134</b>	<b>503</b>	<b>299</b>	<b>22.2</b>	<b>4.73</b>	<b>13.9</b>	<b>1350</b>	<b>251</b>
W18 x 65	133	499	307	14.9	5.97	18.8	1070	248
W12 x 87	132	495	310	5.76	10.8	43.0	740	194
W16 x 67	130	488	307	10.4	8.69	26.1	954	194
W10 x 100	130	488	294	4.01	9.36	57.7	623	226
W21 x 57	129	484	291	20.1	4.77	14.3	1170	256
<b>W21 x 55</b>	<b>126</b>	<b>473</b>	<b>289</b>	<b>16.3</b>	<b>6.11</b>	<b>17.4</b>	<b>1140</b>	<b>234</b>
W14 x 74	126	473	294	8.03	8.76	31.0	795	191
W18 x 60	123	461	284	14.5	5.93	18.2	984	227
W12 x 79	119	446	281	5.67	10.8	39.9	662	175
W14 x 68	115	431	270	7.81	8.69	29.3	722	175
W10 x 88	113	424	259	3.95	9.29	51.1	534	197
<b>W18 x 55</b>	<b>112</b>	<b>420</b>	<b>258</b>	<b>13.9</b>	<b>5.90</b>	<b>17.5</b>	<b>890</b>	<b>212</b>
<b>W21 x 50</b>	<b>110</b>	<b>413</b>	<b>248</b>	<b>18.3</b>	<b>4.59</b>	<b>13.6</b>	<b>984</b>	<b>237</b>
W12 x 72	108	405	256	5.59	10.7	37.4	597	158
<b>W21 x 48</b>	<b>107</b>	<b>398</b>	<b>244</b>	<b>14.7</b>	<b>6.09</b>	<b>16.6</b>	<b>959</b>	<b>217</b>
W16 x 57	105	394	242	12.0	5.56	18.3	758	212
W14 x 61	102	383	242	7.46	8.65	27.5	640	156
W18 x 50	101	379	233	13.1	5.83	17.0	800	192
W10 x 77	97.6	366	225	3.90	9.18	45.2	455	169
W12 x 65	96.8	356	231	5.41	11.9	35.1	533	142
<b>W21 x 44</b>	<b>95.4</b>	<b>358</b>	<b>214</b>	<b>16.8</b>	<b>4.45</b>	<b>13.0</b>	<b>843</b>	<b>217</b>
W16 x 50	92.0	345	213	11.4	5.62	17.2	659	185
W18 x 46	90.7	340	207	14.6	4.56	13.7	712	195
W14 x 53	87.1	327	204	7.93	6.78	22.2	541	155
W12 x 58	86.4	324	205	5.66	8.87	29.9	475	132
W10 x 68	85.3	320	199	3.86	9.15	40.6	394	147
W16 x 45	82.3	309	191	10.8	5.55	16.5	586	167
<b>W18 x 40</b>	<b>78.4</b>	<b>294</b>	<b>180</b>	<b>13.3</b>	<b>4.49</b>	<b>13.1</b>	<b>612</b>	<b>169</b>
W14 x 48	78.4	294	184	7.66	6.75	21.1	484	141
W12 x 53	77.9	292	185	5.48	8.76	28.2	425	125
W10 x 60	74.6	280	175	3.80	9.08	36.6	341	129
<b>W16 x 40</b>	<b>73.0</b>	<b>274</b>	<b>170</b>	<b>10.1</b>	<b>5.55</b>	<b>15.9</b>	<b>518</b>	<b>146</b>
W12 x 50	71.9	270	169	5.97	6.92	23.9	391	135
W8 x 67	70.1	263	159	2.60	7.49	47.7	272	154
W14 x 43	69.6	261	164	7.24	6.68	20.0	428	125
W10 x 54	66.6	250	158	3.74	9.04	33.7	303	112
<b>W18 x 35</b>	<b>66.5</b>	<b>249</b>	<b>151</b>	<b>12.3</b>	<b>4.31</b>	<b>12.4</b>	<b>510</b>	<b>159</b>
W12 x 45	64.2	241	151	5.75	6.89	22.4	348	121
W16 x 36	64.0	240	148	9.31	5.37	15.2	448	140
W14 x 38	61.5	231	143	8.10	5.47	16.2	385	131
W10 x 49	60.4	227	143	3.67	8.97	31.6	272	102
W8 x 58	59.8	224	137	2.56	7.42	41.7	228	134
W12 x 40	57.0	214	135	5.50	6.85	21.1	307	106
W10 x 45	54.9	206	129	3.89	7.10	26.9	248	106





$$M_{rx} = (0.7F_y)S_x$$

$$BF = \frac{M_{px} - M_{rx}}{L_r - L_p}$$

Adapted from *Steel Construction Manual*, 14th ed., AISC, 2011.



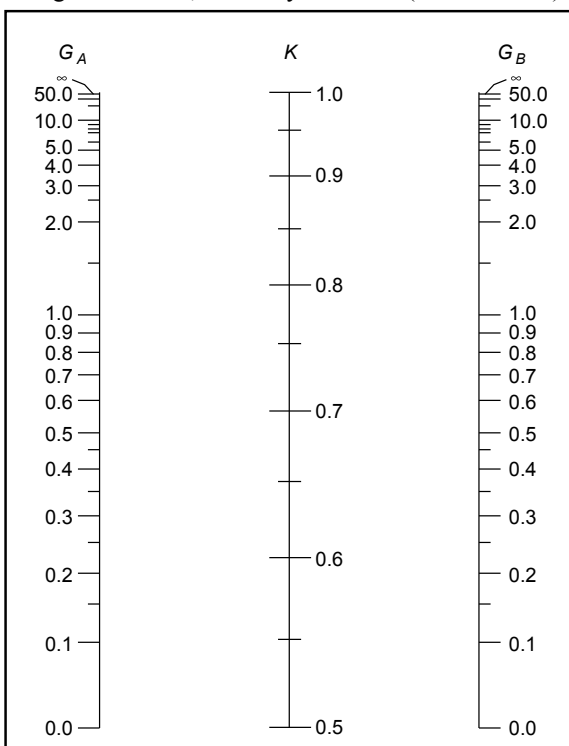


TABLE C-A-7.1 APPROXIMATE VALUES OF EFFECTIVE LENGTH FACTOR, $K$						
BUCKLED SHAPE OF COLUMN IS SHOWN BY DASHED LINE.	(a)	(b)	(c)	(d)	(e)	(f)
THEORETICAL $K$ VALUE	0.5	0.7	1.0	1.0	2.0	2.0
RECOMMENDED DESIGN VALUE WHEN IDEAL CONDITIONS ARE APPROXIMATED	0.65	0.80	1.2	1.0	2.10	2.0
END CONDITION CODE	 ROTATION FIXED AND TRANSLATION FIXED  ROTATION FREE AND TRANSLATION FIXED  ROTATION FIXED AND TRANSLATION FREE  ROTATION FREE AND TRANSLATION FREE					

FOR COLUMN ENDS SUPPORTED BY, BUT NOT RIGIDLY CONNECTED TO, A FOOTING OR FOUNDATION,  $G$  IS THEORETICALLY INFINITY BUT UNLESS DESIGNED AS A TRUE FRICTION-FREE PIN, MAY BE TAKEN AS 10 FOR PRACTICAL DESIGNS. IF THE COLUMN END IS RIGIDLY ATTACHED TO A PROPERLY DESIGNED FOOTING,  $G$  MAY BE TAKEN AS 1.0. SMALLER VALUES MAY BE USED IF JUSTIFIED BY ANALYSIS.

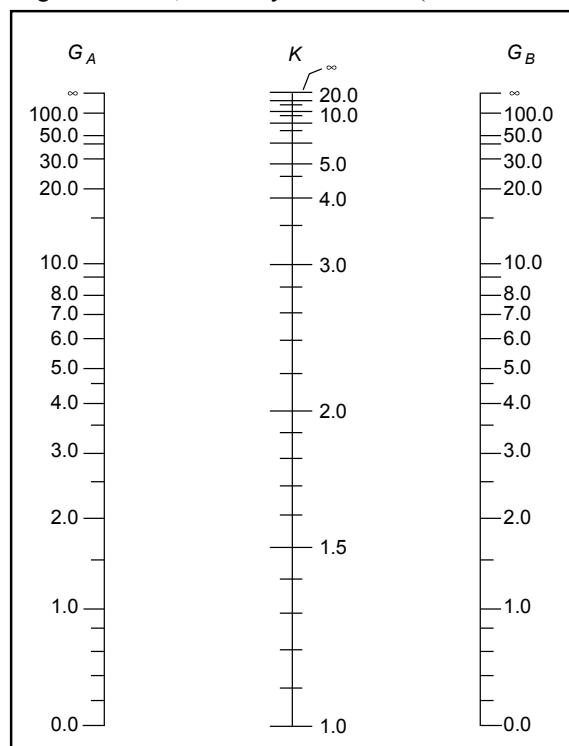
**AISC Figure C-A-7.1**

Alignment chart, sidesway inhibited (braced frame)



**AISC Figure C-A-7.2**


Alignment chart, sidesway uninhibited (moment frame)



AISC Table 4-22  
Available Critical Stress  $\phi_c F_{cr}$  for Compression Members  
 $F_y = 50$  ksi  $\phi_c = 0.90$

$\frac{KL}{r}$	$\phi F_{cr}$ ksi	$\frac{KL}{r}$	$\phi F_{cr}$ ksi	$\frac{KL}{r}$	$\phi F_{cr}$ ksi	$\frac{KL}{r}$	$\phi F_{cr}$ ksi	$\frac{KL}{r}$	$\phi F_{cr}$ ksi
1	45.0	41	39.8	81	27.9	121	15.4	161	8.72
2	45.0	42	39.5	82	27.5	122	15.2	162	8.61
3	45.0	43	39.3	83	27.2	123	14.9	163	8.50
4	44.9	44	39.1	84	26.9	124	14.7	164	8.40
5	44.9	45	38.8	85	26.5	125	14.5	165	8.30
6	44.9	46	38.5	86	26.2	126	14.2	166	8.20
7	44.8	47	38.3	87	25.9	127	14.0	167	8.10
8	44.8	48	38.0	88	25.5	128	13.8	168	8.00
9	44.7	49	37.7	89	25.2	129	13.6	169	7.89
10	44.7	50	37.5	90	24.9	130	13.4	170	7.82
11	44.6	51	37.2	91	24.6	131	13.2	171	7.73
12	44.5	52	36.9	92	24.2	132	13.0	172	7.64
13	44.4	53	36.7	93	23.9	133	12.8	173	7.55
14	44.4	54	36.4	94	23.6	134	12.6	174	7.46
15	44.3	55	36.1	95	23.3	135	12.4	175	7.38
16	44.2	56	35.8	96	22.9	136	12.2	176	7.29
17	44.1	57	35.5	97	22.6	137	12.0	177	7.21
18	43.9	58	35.2	98	22.3	138	11.9	178	7.13
19	43.8	59	34.9	99	22.0	139	11.7	179	7.05
20	43.7	60	34.6	100	21.7	140	11.5	180	6.97
21	43.6	61	34.3	101	21.3	141	11.4	181	6.90
22	43.4	62	34.0	102	21.0	142	11.2	182	6.82
23	43.3	63	33.7	103	20.7	143	11.0	183	6.75
24	43.1	64	33.4	104	20.4	144	10.9	184	6.67
25	43.0	65	33.0	105	20.1	145	10.7	185	6.60
26	42.8	66	32.7	106	19.8	146	10.6	186	6.53
27	42.7	67	32.4	107	19.5	147	10.5	187	6.46
28	42.5	68	32.1	108	19.2	148	10.3	188	6.39
29	42.3	69	31.8	109	18.9	149	10.2	189	6.32
30	42.1	70	31.4	110	18.6	150	10.0	190	6.26
31	41.9	71	31.1	111	18.3	151	9.91	191	6.19
32	41.8	72	30.8	112	18.0	152	9.78	192	6.13
33	41.6	73	30.5	113	17.7	153	9.65	193	6.06
34	41.4	74	30.2	114	17.4	154	9.53	194	6.00
35	41.2	75	29.8	115	17.1	155	9.40	195	5.94
36	40.9	76	29.5	116	16.8	156	9.28	196	5.88
37	40.7	77	29.2	117	16.5	157	9.17	197	5.82
38	40.5	78	28.8	118	16.2	158	9.05	198	5.76
39	40.3	79	28.5	119	16.0	159	8.94	199	5.70
40	40.0	80	28.2	120	15.7	160	8.82	200	5.65

Adapted from *Steel Construction Manual*, 14th ed., AISC, 2011.

<div></div> <div>Selected W14, W12, W10</div>		AISC Table 4–1 Available Strength in Axial Compression, kips—W shapes LRFD: $\phi P_n$												$F_y = 50 \text{ ksi}$ $\phi_c = 0.90$		
Shape wt/ft		W14					W12					W10				
		74	68	61	53	48	58	53	50	45	40	60	54	49	45	39
Effective length KL (ft) with respect to least radius of gyration $r_y$	0	980	899	806	702	636	767	701	657	590	526	794	712	649	597	516
	6	922	844	757	633	573	722	659	595	534	475	750	672	612	543	469
	7	901	826	740	610	552	707	644	574	516	458	734	658	599	525	452
	8	878	804	721	585	529	689	628	551	495	439	717	643	585	505	435
	9	853	781	700	557	504	670	610	526	472	419	698	625	569	483	415
	10	826	755	677	528	477	649	590	499	448	397	677	607	551	460	395
	11	797	728	652	497	449	627	569	471	422	375	655	586	533	435	373
	12	766	700	626	465	420	603	547	443	396	351	631	565	513	410	351
	13	734	670	599	433	391	578	525	413	370	328	606	543	493	384	328
	14	701	639	572	401	361	553	501	384	343	304	581	520	471	358	305
	15	667	608	543	369	332	527	477	354	317	280	555	496	450	332	282
	16	632	576	515	338	304	500	452	326	291	257	528	472	428	306	260
	17	598	544	486	308	276	473	427	297	265	234	501	448	405	281	238
	18	563	512	457	278	250	446	402	270	241	212	474	423	383	256	216
	19	528	480	428	250	224	420	378	244	217	191	447	399	360	233	195
	20	494	448	400	226	202	393	353	220	196	172	420	375	338	210	176
	22	428	387	345	186	167	342	306	182	162	142	367	327	295	174	146
	24	365	329	293	157	140	293	261	153	136	120	317	282	254	146	122
	26	311	281	250	133	120	249	222	130	116	102	270	241	216	124	104
	28	268	242	215	115	103	215	192	112	99.8	88.0	233	208	186	107	90.0
	30	234	211	187	100	89.9	187	167	97.7	87.0	76.6	203	181	162	93.4	78.4
32	205	185	165	88.1		165	147	82.9	76.4	67.3	179	159	143	82.1	68.9	
34	182	164	146			146	130				158	141	126			
36	162	146	130			130	116				141	126	113			
38	146	131	117			117	104				127	113	101			
40	131	119	105			105	93.9				114	102	91.3			

Heavy line indicates KL/r equal to or greater than 200

## HYDROLOGY/WATER RESOURCES

### NRCS (SCS) Rainfall-Runoff

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S}$$

$$S = \frac{1,000}{CN} - 10$$

$$CN = \frac{1,000}{S + 10}$$

$P$  = precipitation (inches)

$S$  = maximum basin retention (inches)

$Q$  = runoff (inches)

$CN$  = curve number

### Rational Formula

$$Q = CIA, \text{ where}$$

$A$  = watershed area (acres)

$C$  = runoff coefficient

$I$  = rainfall intensity (in./hr)

$Q$  = peak discharge (cfs)

### Darcy's Law

$$Q = -KA(dh/dx), \text{ where}$$

$Q$  = discharge rate (ft<sup>3</sup>/sec or m<sup>3</sup>/s)

$K$  = hydraulic conductivity (ft/sec or m/s)

$h$  = hydraulic head (ft or m)

$A$  = cross-sectional area of flow (ft<sup>2</sup> or m<sup>2</sup>)

$$q = -K(dh/dx)$$

$q$  = specific discharge (also called Darcy velocity or superficial velocity)

$$v = q/n = -K/n(dh/dx)$$

$v$  = average seepage velocity

$n$  = effective porosity

**Unit hydrograph:** The direct runoff hydrograph that would result from one unit of runoff occurring uniformly in space and time over a specified period of time.

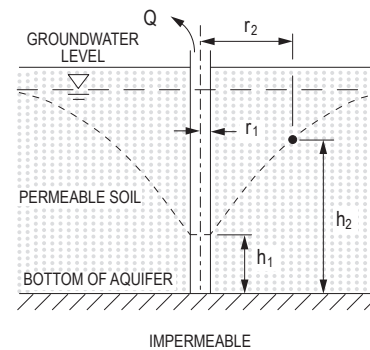
**Transmissivity,  $T$ :** The product of hydraulic conductivity and thickness,  $b$ , of the aquifer ( $L^2T^{-1}$ ).

**Storativity or storage coefficient of an aquifer,  $S$ :**

The volume of water taken into or released from storage per unit surface area per unit change in potentiometric (piezometric) head.

## Well Drawdown

### Unconfined aquifer



### Dupuit's Formula

$$Q = \frac{\pi k (h_2^2 - h_1^2)}{\ln \left( \frac{r_2}{r_1} \right)}$$

where

$Q$  = flow rate of water drawn from well (cfs)

$k$  = coefficient of permeability of soil (fps)

$h_1$  = height of water surface above bottom of aquifer at perimeter of well (ft)

$h_2$  = height of water surface above bottom of aquifer at distance  $r_2$  from well centerline (ft)

$r_1$  = radius to water surface at perimeter of well, i.e., radius of well (ft)

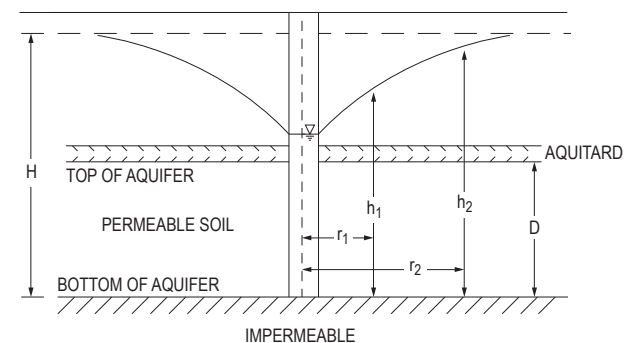
$r_2$  = radius to water surface whose height is  $h_2$  above bottom of aquifer (ft)

$\ln$  = natural logarithm

$Q/D_w$  = specific capacity

$D_w$  = well drawdown (ft)

Confined aquifer:



$$Q = \frac{2\pi T (h_2 - h_1)}{\ln \left( \frac{r_2}{r_1} \right)}$$

where

$T = KD$  = transmissivity (ft<sup>2</sup>/sec)

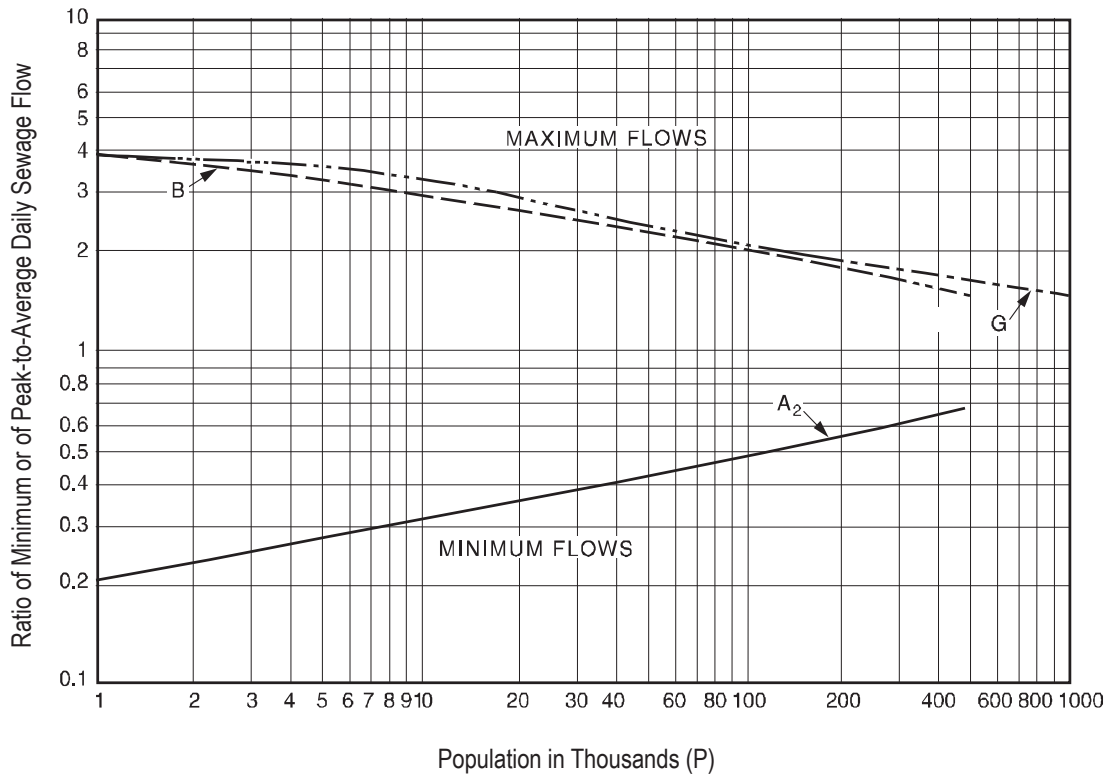
$D$  = thickness of confined aquifer (ft)

$h_1, h_2$  = heights of piezometric surface above bottom of aquifer (ft)

$r_1, r_2$  = radii from pumping well (ft)

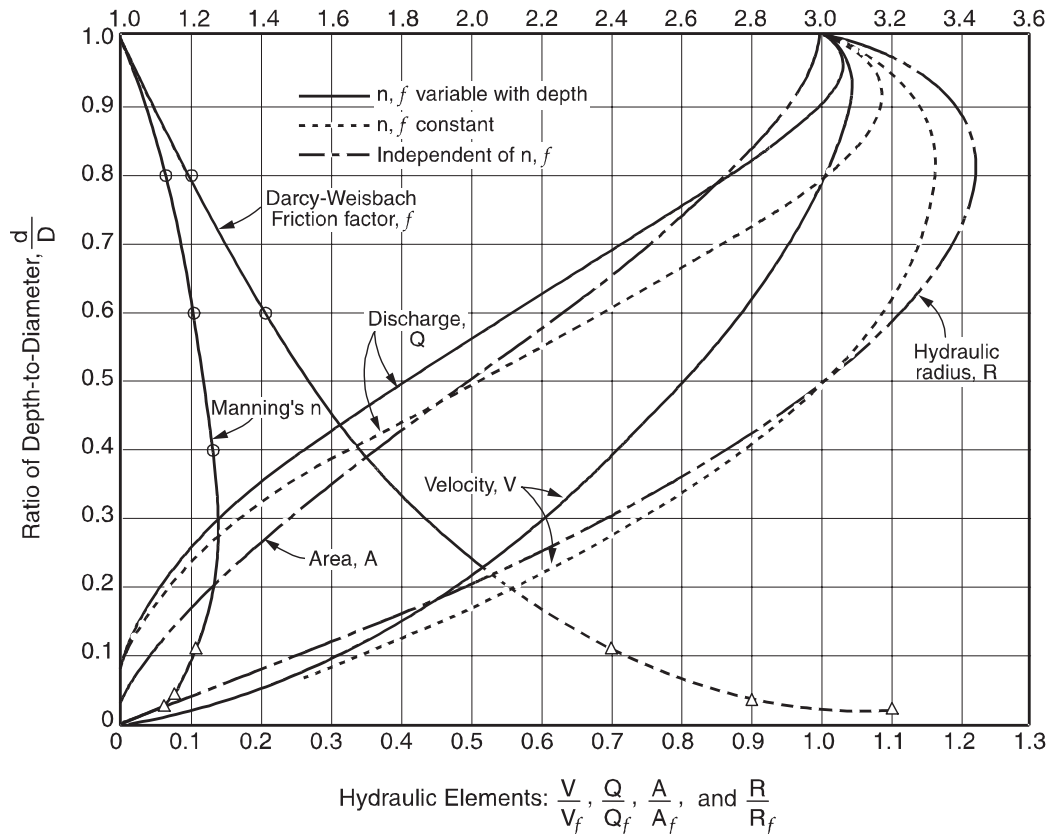
$\ln$  = natural logarithm

## Sewage Flow Ratio Curves



## Hydraulic-Elements Graph for Circular Sewers

Values of:  $\frac{f}{f_f}$  and  $\frac{n}{n_f}$



◆ *Design and Construction of Sanitary and Storm Sewers*, Water Pollution Control Federation and American Society of Civil Engineers, 1970. Reprinted with permission from ASCE. This material may be downloaded from nces.org for personal use only. Any other use requires prior permission of ASCE.

## Open-Channel Flow

### Specific Energy

$$E = \alpha \frac{V^2}{2g} + y = \frac{\alpha Q^2}{2gA^2} + y, \text{ where}$$

$E$  = specific energy

$Q$  = discharge

$V$  = velocity

$y$  = depth of flow

$A$  = cross-sectional area of flow

$\alpha$  = kinetic energy correction factor, usually 1.0

Critical Depth = that depth in a channel at minimum specific energy

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

where  $Q$  and  $A$  are as defined above,

$g$  = acceleration due to gravity

$T$  = width of the water surface

For rectangular channels

$$y_c = \left( \frac{q^2}{g} \right)^{1/3}, \text{ where}$$

$y_c$  = critical depth

$q$  = unit discharge =  $Q/B$

$B$  = channel width

$g$  = acceleration due to gravity

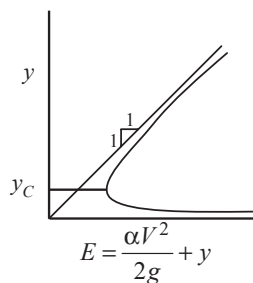
Froude Number = ratio of inertial forces to gravity forces

$$Fr = \frac{V}{\sqrt{gy_h}}, \text{ where}$$

$V$  = velocity

$y_h$  = hydraulic depth =  $A/T$

### Specific Energy Diagram



*Alternate depths:* depths with the same specific energy.

$y_1$  = flow depth at supercritical flow

$y_2$  = flow depth at subcritical flow

$$y_2 = \frac{y_1}{2} (\sqrt{1 + 8 Fr_1^2} - 1)$$

*Uniform flow:* a flow condition where depth and velocity do not change along a channel.

## Manning's Equation

$$Q = \frac{K}{n} A R_H^{2/3} S^{1/2}$$

$Q$  = discharge ( $\text{ft}^3/\text{sec}$  or  $\text{m}^3/\text{s}$ )

$K$  = 1.0 for SI units, 1.486 for USCS units

$A$  = cross-sectional area of flow ( $\text{ft}^2$  or  $\text{m}^2$ )

$R_H$  = hydraulic radius =  $A/P$  ( $\text{ft}$  or  $\text{m}$ )

$P$  = wetted perimeter ( $\text{ft}$  or  $\text{m}$ )

$S$  = slope of hydraulic surface ( $\text{ft}/\text{ft}$  or  $\text{m}/\text{m}$ )

$n$  = Manning's roughness coefficient

Normal depth (uniform flow depth)

$$A R_H^{2/3} = \frac{Qn}{K S^{1/2}}$$

### Weir Formulas

#### Rectangular

Free discharge suppressed

$$Q = CLH^{3/2}$$

Free discharge contracted

$$Q = C(L - 0.2H)H^{3/2}$$

#### V-Notch

$$Q = CH^{5/2}, \text{ where}$$

$Q$  = discharge (cfs or  $\text{m}^3/\text{s}$ )

$C$  = 3.33 for rectangular weir (USCS units)

$C$  = 1.84 for rectangular weir (SI units)

$C$  = 2.54 for  $90^\circ$  V-notch weir (USCS units)

$C$  = 1.40 for  $90^\circ$  V-notch weir (SI units)

$L$  = weir length ( $\text{ft}$  or  $\text{m}$ )

$H$  = head (depth of discharge over weir)  $\text{ft}$  or  $\text{m}$

### Hazen-Williams Equation

$$V = k_1 C R_H^{0.63} S^{0.54}, \text{ where}$$

$C$  = roughness coefficient

$k_1$  = 0.849 for SI units

$k_1$  = 1.318 for USCS units

$R_H$  = hydraulic radius ( $\text{ft}$  or  $\text{m}$ )

$S$  = slope of energy grade line

$$= h_f/L \text{ (ft/ft or m/m)}$$

$V$  = velocity ( $\text{ft}/\text{sec}$  or  $\text{m}/\text{s}$ )

### Circular Pipe Head Loss Equation (Head Loss Expressed in Feet)

$$h_f = \frac{4.73 L}{C^{1.852} D^{4.87}} Q^{1.852}, \text{ where}$$

- $h_f$  = head loss (ft)  
 $L$  = pipe length (ft)  
 $D$  = pipe diameter (ft)  
 $Q$  = flow (cfs)  
 $C$  = Hazen-Williams coefficient

### Circular Pipe Head Loss Equation (Head Loss Expressed as Pressure)

#### U.S. Customary Units

$$P = \frac{4.52 Q^{1.85}}{C^{1.85} D^{4.87}}, \text{ where}$$

- $P$  = pressure loss (psi per foot of pipe)  
 $Q$  = flow (gpm)  
 $D$  = pipe diameter (inches)  
 $C$  = Hazen-Williams coefficient

#### SI Units

$$P = \frac{6.05 Q^{1.85}}{C^{1.85} D^{4.87}} \times 10^5, \text{ where}$$

- $P$  = pressure loss (bars per meter of pipe)  
 $Q$  = flow (liters/minute)  
 $D$  = pipe diameter (mm)

#### Values of Hazen-Williams Coefficient $C$

Pipe Material	$C$
Ductile iron	140
Concrete (regardless of age)	130
Cast iron:	
New	130
5 yr old	120
20 yr old	100
Welded steel, new	120
Wood stave (regardless of age)	120
Vitrified clay	110
Riveted steel, new	110
Brick sewers	100
Asbestos-cement	140
Plastic	150

### TRANSPORTATION

#### U.S. Customary Units

- $a$  = deceleration rate (ft/sec<sup>2</sup>)  
 $A$  = absolute value of algebraic difference in grades (%)  
 $e$  = superelevation (%)  
 $f$  = side friction factor  
 $\pm G$  = percent grade divided by 100 (uphill grade "+")  
 $h_1$  = height of driver's eyes above the roadway surface (ft)  
 $h_2$  = height of object above the roadway surface (ft)  
 $L$  = length of curve (ft)  
 $L_s$  = spiral transition length (ft)  
 $R$  = radius of curve (ft)  
 $SSD$  = stopping sight distance (ft)  
 $t$  = driver reaction time (sec)  
 $V$  = design speed (mph)  
 $v$  = vehicle approach speed (fps)  
 $W$  = width of intersection, curb-to-curb (ft)  
 $l$  = length of vehicle (ft)  
 $y$  = length of yellow interval to nearest 0.1 sec (sec)  
 $r$  = length of red clearance interval to nearest 0.1 sec (sec)

#### Vehicle Signal Change Interval

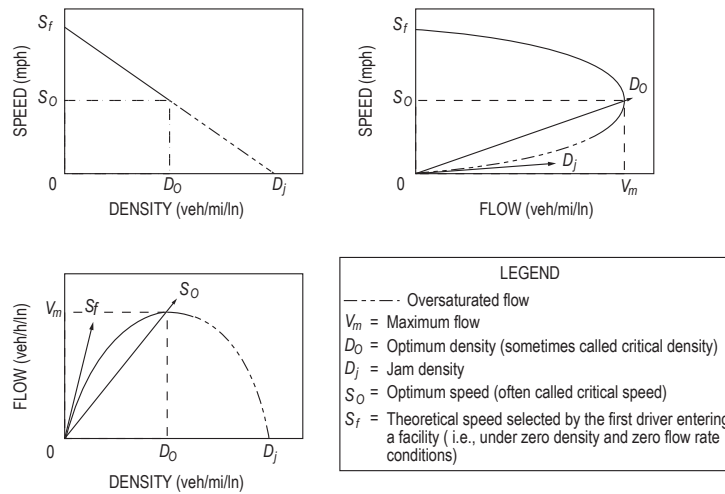
$$y = t + \frac{v}{2a \pm 64.4 G}$$

$$r = \frac{W + l}{v}$$

#### Stopping Sight Distance

$$SSD = 1.47 Vt + \frac{V^2}{30 \left( \left( \frac{a}{32.2} \right) \pm G \right)}$$

## Traffic Flow Relationships ( $q = kv$ )



Vertical Curves: Sight Distance Related to Curve Length		
	$S \leq L$	$S > L$
Crest Vertical Curve General equation:	$L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2}$	$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A}$
Standard Criteria: $h_1 = 3.50$ ft and $h_2 = 2.0$ ft:	$L = \frac{AS^2}{2,158}$	$L = 2S - \frac{2,158}{A}$
Sag Vertical Curve (based on standard headlight criteria)	$L = \frac{AS^2}{400 + 3.5S}$	$L = 2S - \left( \frac{400 + 3.5S}{A} \right)$
Sag Vertical Curve (based on riding comfort)	$L = \frac{AV^2}{46.5}$	
Sag Vertical Curve (based on adequate sight distance under an overhead structure to see an object beyond a sag vertical curve)	$L = \frac{AS^2}{800 \left( C - \frac{h_1 + h_2}{2} \right)}$	$L = 2S - \frac{800}{A} \left( C - \frac{h_1 + h_2}{2} \right)$
$C$ = vertical clearance for overhead structure (overpass) located within 200 feet of the midpoint of the curve		

Horizontal Curves	
Side friction factor (based on superelevation)	$0.01e + f = \frac{V^2}{15R}$
Spiral Transition Length	$L_s = \frac{3.15V^3}{RC}$ <p><math>C</math> = rate of increase of lateral acceleration [use 1 ft/sec<sup>3</sup> unless otherwise stated]</p>
Sight Distance (to see around obstruction)	$HSO = R \left[ 1 - \cos \left( \frac{28.65S}{R} \right) \right]$ <p>HSO = Horizontal sight line offset</p>

- ◆ AASHTO, *A Policy on Geometric Design of Highways and Streets*, 6th ed., 2011. Used by permission.
- Compiled from AASHTO, *A Policy on Geometric Design of Highways and Streets*, 6th ed., 2011.



### Horizontal Curve Formulas

- $D$  = Degree of Curve, Arc Definition  
 $PC$  = Point of Curve (also called  $BC$ )  
 $PT$  = Point of Tangent (also called  $EC$ )  
 $PI$  = Point of Intersection  
 $I$  = Intersection Angle (also called  $\Delta$ )  
     Angle Between Two Tangents  
 $L$  = Length of Curve, from  $PC$  to  $PT$   
 $T$  = Tangent Distance  
 $E$  = External Distance  
 $R$  = Radius  
 $LC$  = Length of Long Chord  
 $M$  = Length of Middle Ordinate  
 $c$  = Length of Sub-Chord  
 $d$  = Angle of Sub-Chord  
 $l$  = Curve Length for Sub-Chord

$$R = \frac{5729.58}{D}$$

$$R = \frac{LC}{2 \sin(I/2)}$$

$$T = R \tan(I/2) = \frac{LC}{2 \cos(I/2)}$$

$$L = RI \frac{\pi}{180} = \frac{I}{D} 100$$

$$M = R[1 - \cos(I/2)]$$

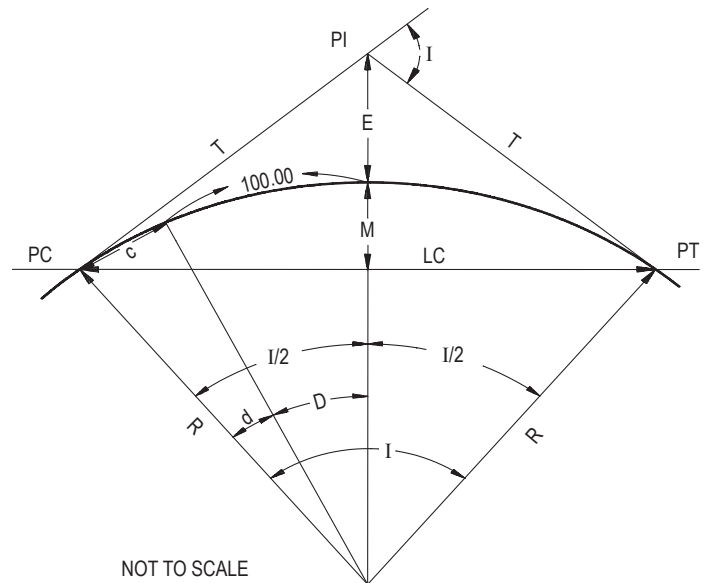
$$\frac{R}{E + R} = \cos(I/2)$$

$$\frac{R - M}{R} = \cos(I/2)$$

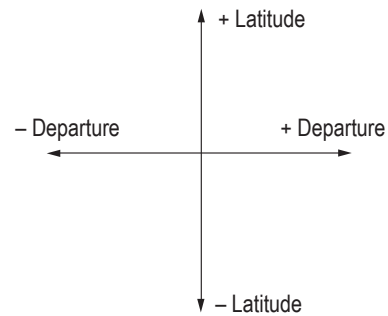
$$c = 2R \sin(d/2)$$

$$l = Rd \left( \frac{\pi}{180} \right)$$

$$E = R \left[ \frac{1}{\cos(I/2)} - 1 \right]$$

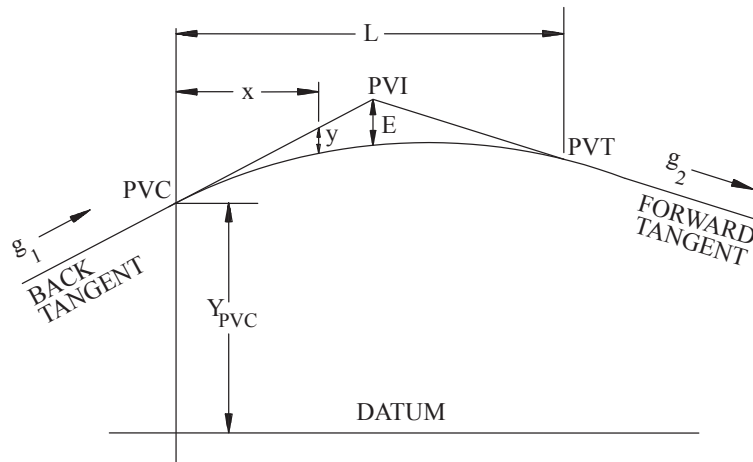


### LATITUDES AND DEPARTURES



Deflection angle per 100 feet of arc length equals  $D/2$

## Vertical Curve Formulas



VERTICAL CURVE FORMULAS  
NOT TO SCALE

$L$  = Length of curve

$PVC$  = Point of vertical curvature

$PVI$  = Point of vertical intersection

$PVT$  = Point of vertical tangency

$g_1$  = Grade of back tangent

$x$  = Horizontal distance from  $PVC$   
to point on curve

$g_2$  = Grade of forward tangent

$a$  = Parabola constant

$y$  = Tangent offset

$E$  = Tangent offset at  $PVI = AL/800$

$r$  = Rate of change of grade

$K$  = Rate of vertical curvature

$$x_m = \text{Horizontal distance to min/max elevation on curve} = -\frac{g_1}{2a} = \frac{g_1 L}{g_1 - g_2}$$

$$\text{Tangent elevation} = Y_{PVC} + g_1 x \text{ and } = Y_{PVI} + g_2 (x - L/2)$$

$$\text{Curve elevation} = Y_{PVC} + g_1 x + ax^2 = Y_{PVC} + g_1 x + [(g_2 - g_1)/(2L)]x^2$$

$$y = ax^2 \quad a = \frac{g_2 - g_1}{2L} \quad E = a\left(\frac{L}{2}\right)^2 \quad r = \frac{g_2 - g_1}{L} \quad K = \frac{L}{A}$$

## EARTHWORK FORMULAS

Average End Area Formula,  $V = L(A_1 + A_2)/2$

Prismoidal Formula,  $V = L(A_1 + 4A_m + A_2)/6$

where  $A_m$  = area of mid-section

$L$  = distance between  $A_1$  and  $A_2$

Pyramid or Cone,  $V = h(\text{Area of Base})/3$

## AREA FORMULAS

Area by Coordinates:  $\text{Area} = [X_A(Y_B - Y_N) + X_B(Y_C - Y_A) + X_C(Y_D - Y_B) + \dots + X_N(Y_A - Y_{N-1})]/2$

$$\text{Trapezoidal Rule: Area} = w\left(\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4 + \dots + h_{n-1}\right)$$

$w$  = common interval

$$\text{Simpson's 1/3 Rule: Area} = w\left[h_1 + 2\left(\sum_{k=3,5,\dots}^{n-2} h_k\right) + 4\left(\sum_{k=2,4,\dots}^{n-1} h_k\right) + h_n\right]/3$$

$n$  must be odd number of measurements  
(only for Simpson's 1/3 Rule)

## Highway Pavement Design

AASHTO Structural Number Equation	
$SN = a_1D_1 + a_2D_2 + \dots + a_nD_n$ , where $SN$ = structural number for the pavement $a_i$ = layer coefficient and $D_i$ = thickness of layer (inches).	

Gross Axle Load		Load Equivalency Factors		Gross Axle Load		Load Equivalency Factors	
kN	lb	Single Axles	Tandem Axles	kN	lb	Single Axles	Tandem Axles
4.45	1,000	0.00002		187.0	42,000	25.64	2.51
8.9	2,000	0.00018		195.7	44,000	31.00	3.00
17.8	4,000	0.00209		200.0	45,000	34.00	3.27
22.25	5,000	0.00500		204.5	46,000	37.24	3.55
<b>26.7</b>	<b>6,000</b>	<b>0.01043</b>		<b>213.5</b>	<b>48,000</b>	<b>44.50</b>	<b>4.17</b>
35.6	8,000	0.0343		222.4	50,000	52.88	4.86
44.5	10,000	0.0877	0.00688	231.3	52,000		5.63
53.4	12,000	0.189	0.0144	240.2	54,000		6.47
62.3	14,000	0.360	0.0270	244.6	55,000		6.93
<b>66.7</b>	<b>15,000</b>	<b>0.478</b>	<b>0.0360</b>	<b>249.0</b>	<b>56,000</b>		<b>7.41</b>
71.2	16,000	0.623	0.0472	258.0	58,000		8.45
80.0	18,000	1.000	0.0773	267.0	60,000		9.59
89.0	20,000	1.51	0.1206	275.8	62,000		10.84
97.8	22,000	2.18	0.180	284.5	64,000		12.22
<b>106.8</b>	<b>24,000</b>	<b>3.03</b>	<b>0.260</b>	<b>289.0</b>	<b>65,000</b>		<b>12.96</b>
111.2	25,000	3.53	0.308	293.5	66,000		13.73
115.6	26,000	4.09	0.364	302.5	68,000		15.38
124.5	28,000	5.39	0.495	311.5	70,000		17.19
133.5	30,000	6.97	0.658	320.0	72,000		19.16
<b>142.3</b>	<b>32,000</b>	<b>8.88</b>	<b>0.857</b>	<b>329.0</b>	<b>74,000</b>		<b>21.32</b>
151.2	34,000	11.18	1.095	333.5	75,000		22.47
155.7	35,000	12.50	1.23	338.0	76,000		23.66
160.0	36,000	13.93	1.38	347.0	78,000		26.22
169.0	38,000	17.20	1.70	356.0	80,000		28.99
<b>178.0</b>	<b>40,000</b>	<b>21.08</b>	<b>2.08</b>				
Note: kN converted to lb are within 0.1 percent of lb shown							

## Superpave

### PERFORMANCE-GRADED (PG) BINDER GRADING SYSTEM

PERFORMANCE GRADE	PG 52							PG 58					PG 64				
	-10	-16	-22	-28	-34	-40	-46	-16	-22	-28	-34	-40	-16	-22	-28	-34	-40
AVERAGE 7-DAY MAXIMUM PAVEMENT DESIGN TEMPERATURE, °C <sup>a</sup>	<52							<58					<64				
MINIMUM PAVEMENT DESIGN TEMPERATURE, °C <sup>a</sup>	>-10	>-16	>-22	>-28	>-34	>-40	>-46	>-16	>-22	>-28	>-34	>-40	>-16	>-22	>-28	>-34	>-40
ORIGINAL BINDER																	
FLASH POINT TEMP, T48: MINIMUM °C	230																
VISCOSITY, ASTM D 4402: <sup>b</sup> MAXIMUM, 3 Pa-s (3,000 cP), TEST TEMP, °C	135																
DYNAMIC SHEAR, TP5: <sup>c</sup> G*/sin δ , MINIMUM, 1.00 kPa TEST TEMPERATURE @ 10 rad/sec., °C	52							58					64				
ROLLING THIN FILM OVEN (T240) OR THIN FILM OVEN (T179) RESIDUE																	
MASS LOSS, MAXIMUM, %	1.00																
DYNAMIC SHEAR, TP5: G*/sin δ , MINIMUM, 2.20 kPa TEST TEMP @ 10 rad/sec. °C	52							58					64				
PRESSURE AGING VESSEL RESIDUE (PP1)																	
PAV AGING TEMPERATURE, °C <sup>d</sup>	90							100					100				
DYNAMIC SHEAR, TP5: G*/sin δ , MAXIMUM, 5,000 kPa TEST TEMP @ 10 rad/sec. °C	25	22	19	16	13	10	7	25	22	19	16	13	28	25	22	19	16
PHYSICAL HARDENING <sup>e</sup>	REPORT																
CREEP STIFFNESS, TP1: <sup>f</sup> S, MAXIMUM, 300 MPa M-VALUE, MINIMUM, 0.300 TEST TEMP, @ 60 sec., °C	0	-6	-12	-18	-24	-30	-36	-6	-12	-18	-24	-30	-6	-12	-18	-24	-30
DIRECT TENSION, TP3: <sup>f</sup> FAILURE STRAIN, MINIMUM, 1.0% TEST TEMP @ 1.0 mm/min, °C	0	-6	-12	-18	-24	-30	-36	-6	-12	-18	-24	-30	-6	-12	-18	-24	-30

Federal Highway Administration Report FHWA-SA-95-03, "Background of Superpave Asphalt Mixture Design and Analysis," Nov. 1994.

## Superpave Mixture Design: Compaction Requirements

SUPERPAVE GYRATORY COMPACTION EFFORT												
TRAFFIC, MILLION ESALS	AVERAGE DESIGN HIGH AIR TEMPERATURE											
	< 39°C			39° – 40°C			41° – 42°C			42° – 43°C		
	N <sub>int</sub>	N <sub>des</sub>	N <sub>max</sub>	N <sub>int</sub>	N <sub>des</sub>	N <sub>max</sub>	N <sub>int</sub>	N <sub>des</sub>	N <sub>max</sub>	N <sub>int</sub>	N <sub>des</sub>	N <sub>max</sub>
< 0.3	7	68	104	7	74	114	7	78	121	7	82	127
< 1	7	76	117	7	83	129	7	88	138	8	93	146
< 3	7	86	134	8	95	150	8	100	158	8	105	167
< 10	8	96	152	8	106	169	8	113	181	9	119	192
< 30	8	109	174	9	121	195	9	128	208	9	135	220
< 100	9	126	204	9	139	228	9	146	240	10	153	253
≥ 100	9	142	233	10	158	262	10	165	275	10	177	288

VFA REQUIREMENTS @ 4% AIR VOIDS	
TRAFFIC, MILLION ESALS	DESIGN VFA (%)
< 0.3	70 – 80
< 1	65 – 78
< 3	65 – 78
< 10	65 – 75
< 30	65 – 75
< 100	65 – 75
≥ 100	65 – 75

VMA REQUIREMENTS @ 4% AIR VOIDS					
NOMINAL MAXIMUM AGGREGATE SIZE (mm)	9.5	12.5	19.0	25.0	37.5
MINIMUM VMA (%)	15	14	13	12	11

COMPACTION KEY			
SUPERPAVE GYRATORY COMPACTION	N <sub>int</sub>	N <sub>des</sub>	N <sub>max</sub>
PERCENT OF Gmm	≤ 89%	96%	≤ 98%

Federal Highway Administration Report FHWA-SA-95-03, "Background of Superpave Asphalt Mixture Design and Analysis," Nov. 1994.

### Gravity Model

$$T_{ij} = P_i \left[ \frac{A_j F_{ij} K_{ij}}{\sum_j A_j F_{ij} K_{ij}} \right]$$

where

$T_{ij}$  = number of trips that are produced in zone  $i$  and attracted to zone  $j$

$P_i$  = total number of trips produced in zone  $i$

$A_j$  = number of trips attracted to zone  $j$

$F_{ij}$  = a value that is an inverse function of travel time between zones  $i$  and  $j$

$K_{ij}$  = socioeconomic adjustment factor for interchange  $ij$

### Logit Models

$$U_x = \sum_{i=1}^n a_i X_i$$

where

$U_x$  = utility of mode  $x$

$n$  = number of attributes

$X_i$  = attribute value (time, cost, and so forth)

$a_i$  = coefficient value for attributes  $i$  (negative, since the values are disutilities)

If two modes, auto ( $A$ ) and transit ( $T$ ), are being considered, the probability of selecting the auto mode  $A$  can be written as

$$P(A) = \frac{e^{U_A}}{e^{U_A} + e^{U_T}} \quad P(x) = \frac{e^{U_x}}{\sum_{e=1}^n e^{U_{xi}}}$$

## Traffic Safety Equations

$$RMEV = \frac{A \times 1,000,000}{V}$$

where

$RMEV$  = crash rate per million entering vehicles

$A$  = number of crashes, total or by type occurring in a single year at the location

$V$  =  $ADT \times 365$

$ADT$  = average daily traffic entering intersection

$$RMVM = \frac{A \times 100,000,000}{VMT}$$

where

$RMVM$  = crash rate per million vehicle miles

$A$  = number of crashes, total or by type at the study location, during a given period

$VMT$  = vehicle miles of travel during the given period

=  $ADT \times (\text{number of days in study period}) \times (\text{length of road})$

$$\text{Crashes prevented} = N \times CR \frac{(ADT \text{ after improvement})}{(ADT \text{ before improvement})}$$

where

$N$  = expected number of crashes if countermeasure is not implemented and if the traffic volume remains the same

$CR = CR_1 + (1 - CR_1)CR_2 + (1 - CR_1)(1 - CR_2)CR_3 + \dots + (1 - CR_1) \dots (1 - CR_{m-1}) CR_m$   
= overall crash reduction factor for multiple mutually exclusive improvements at a single site

$CR_i$  = crash reduction factor for a specific countermeasure  $i$

$m$  = number of countermeasures at the site

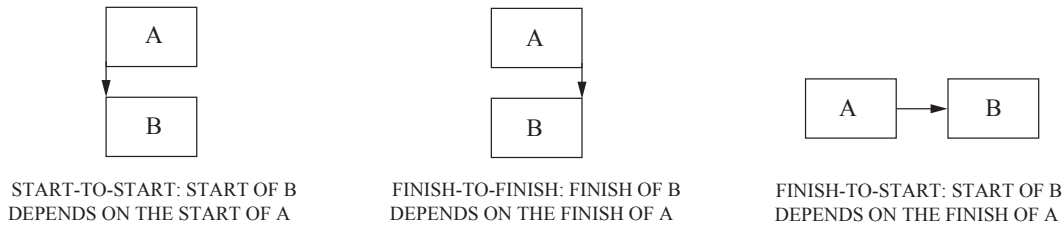
Garber, Nicholas J., and Lester A. Hoel, *Traffic and Highway Engineering*, 4th ed., Cengage Learning, 2009.

## CONSTRUCTION

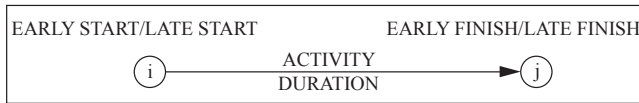
Construction project scheduling and analysis questions may be based on either the activity-on-node method or the activity-on-arrow method.

### CPM PRECEDENCE RELATIONSHIPS

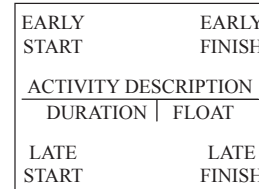
#### ACTIVITY-ON-NODE



#### ACTIVITY-ON-ARROW ANNOTATION



#### ACTIVITY-ON-NODE ANNOTATION



### Nomenclature

ES = Early start = Latest EF of predecessors

EF = Early finish = ES + duration

LS = Late start = LF – duration

LF = Late finish = Earliest LS of successors

D = Duration

Float = LS – ES or LF – EF

### Determining the Project Size Modifier

Square Foot Base Size							
Building Type	Median Cost per S.F.	Typical Size Gross S.F.	Typical Range Gross S.F.	Building Type	Median Cost per S.F.	Typical Size Gross S.F.	Typical Range Gross S.F.
Apartments, Low Rise	\$ 54.05	21,000	9,700–37,200	Jails	\$ 165.00	13,700	7,500–28,000
Apartments, Mid Rise	68.25	50,000	32,000–100,000	Libraries	97.30	12,000	7,000–31,000
Apartments, High Rise	78.30	310,000	100,000–650,000	Medical Clinics	93.15	7,200	4,200–15,700
Auditoriums	90.35	25,000	7,600–39,000	Medical Offices	87.50	6,000	4,000–15,000
Auto Sales	55.90	20,000	10,800–28,600	Motels	67.00	27,000	15,800–51,000
Banks	121.00	4,200	2,500–7,500	Nursing Homes	89.95	23,000	15,000–37,000
Churches	81.60	9,000	5,300–13,200	Offices, Low Rise	73.00	8,600	4,700–19,000
Clubs, Country	81.40	6,500	4,500–15,000	Offices, Mid Rise	76.65	52,000	31,300–83,100
Clubs, Social	79.15	10,000	6,000–13,500	Offices, High Rise	98.05	260,000	151,000–468,000
Clubs, YMCA	81.60	28,300	12,800–39,400	Police Stations	122.00	10,500	4,000–19,000
Colleges (Class)	107.00	50,000	23,500–98,500	Post Offices	90.40	12,400	6,800–30,000
Colleges (Science Lab)	156.00	45,600	16,600–80,000	Power Plants	678.00	7,500	1,000–20,000
College (Student Union)	119.00	33,400	16,000–85,000	Religious Education	74.85	9,000	6,000–12,000
Community Center	85.05	9,400	5,300–16,700	Research	127.00	19,000	6,300–45,000
Court Houses	116.00	32,400	17,800–106,000	Restaurants	110.00	4,400	2,800–6,000
Dept. Stores	50.50	90,000	44,000–122,000	Retail Stores	53.70	7,200	4,000–17,600
Dormitories, Low Rise	87.20	24,500	13,400–40,000	Schools, Elementary	78.20	41,000	24,500–55,000
Dormitories, Mid Rise	113.00	55,600	36,100–90,000	Schools, Jr. High	79.65	92,000	52,000–119,000
Factories	48.95	26,400	12,900–50,000	Schools, Sr. High	79.65	101,000	50,500–175,000
Fire Stations	85.45	5,800	4,000–8,700	Schools, Vocational	79.35	37,000	20,500–82,000
Fraternity Houses	84.10	12,500	8,200–14,800	Sports Arenas	66.45	15,000	5,000–40,000
Funeral Homes	94.00	7,800	4,500–11,000	Supermarkets	53.85	20,000	12,000–30,000
Garages, Commercial	59.70	9,300	5,000–13,600	Swimming Pools	125.00	13,000	7,800–22,000
Garages, Municipal	76.40	8,300	4,500–12,600	Telephone Exchange	145.00	4,500	1,200–10,600
Garages, Parking	31.30	163,000	76,400–225,300	Theaters	79.70	10,500	8,800–17,500
Gymnasiums	78.95	19,200	11,600–41,000	Town Halls	87.65	10,800	4,800–23,400
Hospitals	149.00	55,000	27,200–125,000	Warehouses	36.15	25,000	8,000–72,000
House (Elderly)	73.90	37,000	21,000–66,000	Warehouse and Office	41.75	25,000	8,000–72,000
Housing (Public)	68.45	36,000	14,400–74,400				
Ice Rinks	76.00	29,000	27,200–33,600				

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**Earned-Value Analysis**

BCWS = Budgeted cost of work scheduled (Planned)

ACWP = Actual cost of work performed (Actual)

BCWP = Budgeted cost of work performed (Earned)

**Variances**

CV = BCWP – ACWP (Cost variance = Earned – Actual)

SV = BCWP – BCWS (Schedule variance = Earned – Planned)

**Indices**

$CPI = \frac{BCWP}{ACWP}$  (Cost Performance Index =  $\frac{\text{Earned}}{\text{Actual}}$ )

$SPI = \frac{BCWP}{BCWS}$  (Schedule Performance Index =  $\frac{\text{Earned}}{\text{Planned}}$ )

**Forecasting**

BAC = Original project estimate (Budget at completion)

$ETC = \frac{BAC - BCWP}{CPI}$  (Estimate to complete)

$EAC = (ACWP + ETC)$  (Estimate to complete)



# ENVIRONMENTAL ENGINEERING

## AIR POLLUTION

### Nomenclature

$$\frac{\mu\text{g}}{\text{m}^3} = \text{ppb} \times \frac{P(\text{MW})}{RT}$$

ppb = parts per billion

$P$  = pressure (atm)

$R$  = ideal gas law constant  
= 0.0821 L·atm/(mole·K)

$T$  = absolute temperature, K = 273.15 + °C

MW = molecular weight (g/mole)

### Atmospheric Dispersion Modeling (Gaussian)

$\sigma_y$  and  $\sigma_z$  as a function of downwind distance and stability class, see following figures.

$$C = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2} \frac{y^2}{\sigma_y^2}\right) \left[ \exp\left(-\frac{1}{2} \frac{(z-H)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2} \frac{(z+H)^2}{\sigma_z^2}\right) \right]$$

where

$C$  = steady-state concentration at a point ( $x, y, z$ ) ( $\mu\text{g}/\text{m}^3$ )

$Q$  = emissions rate ( $\mu\text{g}/\text{s}$ )

$\sigma_y$  = horizontal dispersion parameter (m)

$\sigma_z$  = vertical dispersion parameter (m)

$u$  = average wind speed at stack height (m/s)

$y$  = horizontal distance from plume centerline (m)

$z$  = vertical distance from ground level (m)

$H$  = effective stack height (m) =  $h + \Delta h$

where  $h$  = physical stack height

$\Delta h$  = plume rise

$x$  = downwind distance along plume centerline (m)

Maximum concentration at ground level and directly downwind from an elevated source.

$$C_{\max} = \frac{Q}{\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2} \frac{(H^2)}{\sigma_z^2}\right)$$

where variables are as above except

$C_{\max}$  = maximum ground-level concentration

$\sigma_z = \frac{H}{\sqrt{2}}$  for neutral atmospheric conditions

### Selected Properties of Air

Nitrogen ( $\text{N}_2$ ) by volume	78.09%
Oxygen ( $\text{O}_2$ ) by volume	20.94%
Argon (Ar) by volume	0.93%
Molecular weight of air	28.966 g/mol
Absolute viscosity, $\mu$	
at 80°F	0.045 lbm/(hr-ft)
at 100°F	0.047 lbm/(hr-ft)
Density	
at 80°F	0.0734 lbm/ft <sup>3</sup>
at 100°F	0.0708 lbm/ft <sup>3</sup>

The dry adiabatic lapse rate  $\Gamma_{\text{AD}}$  is 0.98°C per 100 m (5.4°F per 1,000 ft). This is the rate at which dry air cools adiabatically with altitude. The actual (environmental) lapse rate  $\Gamma$  is compared to  $\Gamma_{\text{AD}}$  to determine stability as follows:

Lapse Rate	Stability Condition
$\Gamma > \Gamma_{\text{AD}}$	Unstable
$\Gamma = \Gamma_{\text{AD}}$	Neutral
$\Gamma < \Gamma_{\text{AD}}$	Stable

### Atmospheric Stability Under Various Conditions

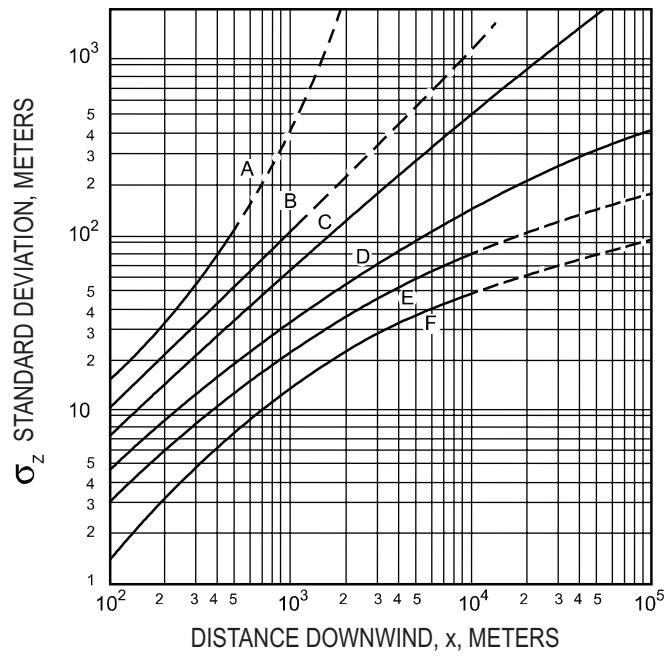
Surface Wind Speed <sup>a</sup> (m/s)	Day Solar Insolation			Night Cloudiness <sup>e</sup>	
	Strong <sup>b</sup>	Moderate <sup>c</sup>	Slight <sup>d</sup>	Cloudy (≥4/8)	Clear (≤3/8)
<2	A	A-B <sup>f</sup>	B	E	F
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
>6	C	D	D	D	D

**Notes:**

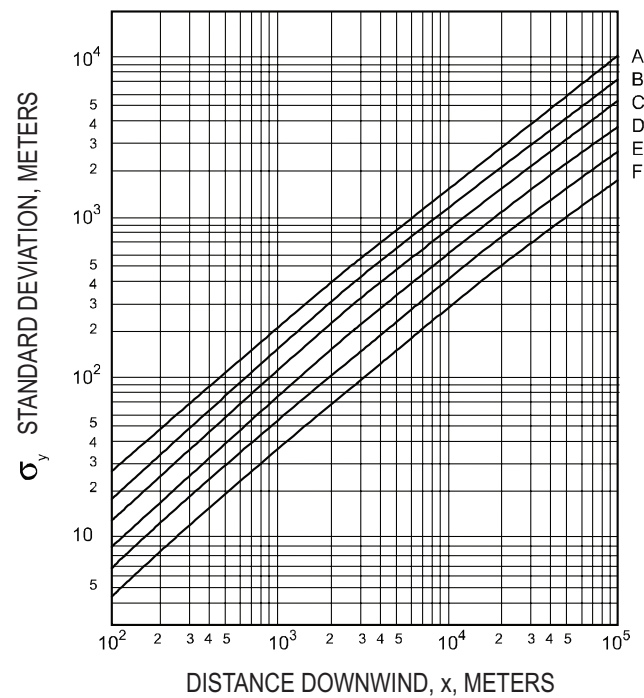
- a. Surface wind speed is measured at 10 m above the ground.
  - b. Corresponds to clear summer day with sun higher than 60° above the horizon.
  - c. Corresponds to a summer day with a few broken clouds, or a clear day with sun 35-60° above the horizon.
  - d. Corresponds to a fall afternoon, or a cloudy summer day, or clear summer day with the sun 15-35°.
  - e. Cloudiness is defined as the fraction of sky covered by the clouds.
  - f. For A-B, B-C, or C-D conditions, average the values obtained for each.
- \* A = Very unstable                      D = Neutral  
      B = Moderately unstable            E = Slightly stable  
      C = Slightly unstable                F = Stable

Regardless of wind speed, Class D should be assumed for overcast conditions, day or night.

Turner, D.B., "Workbook of Atmospheric Dispersion Estimates: An Introduction to Dispersion Modeling," 2nd ed., Lewis Publishing/CRC Press, Florida, 1994.



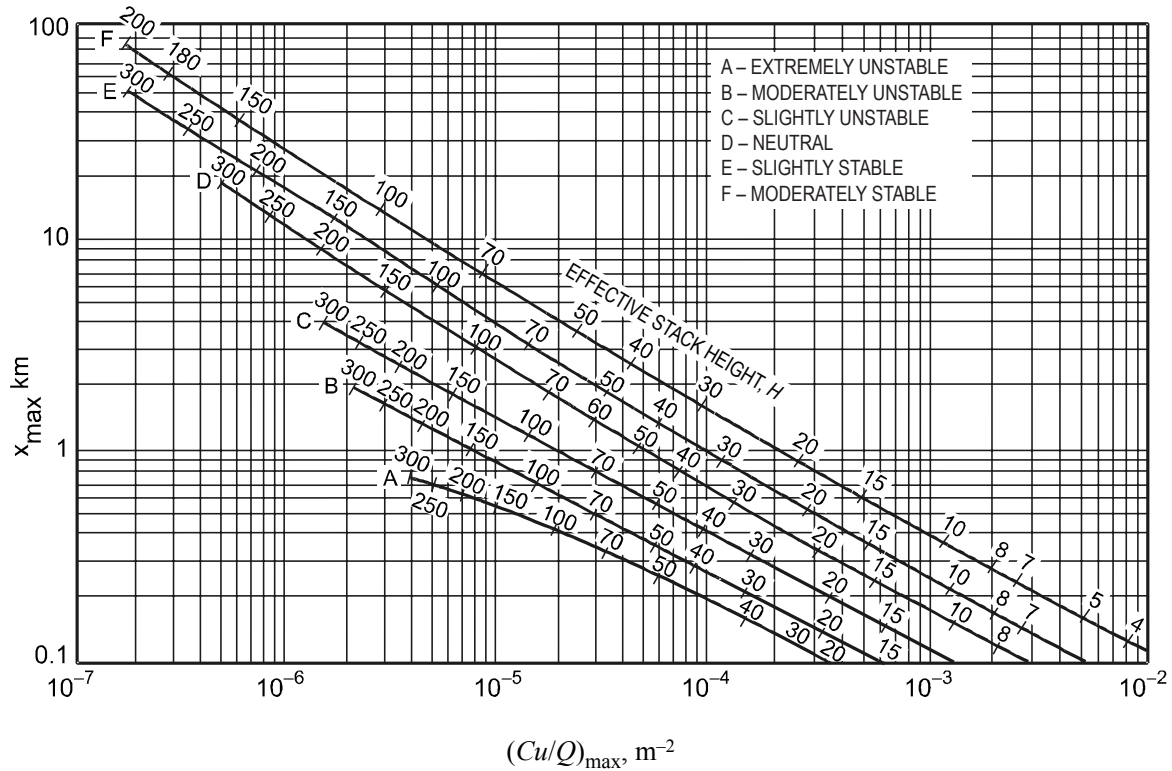
**VERTICAL STANDARD DEVIATIONS OF A PLUME**



**HORIZONTAL STANDARD DEVIATIONS OF A PLUME**

- A – EXTREMELY UNSTABLE
- B – MODERATELY UNSTABLE
- C – SLIGHTLY UNSTABLE
- D – NEUTRAL
- E – SLIGHTLY STABLE
- F – MODERATELY STABLE

Downwind distance where the maximum concentration occurs,  $x_{\max}$ , versus  $(Cu/Q)_{\max}$  as a function of stability class



**NOTES:** Effective stack height shown on curves numerically.

$x_{\max}$  = distance along plume centerline to the point of maximum concentration

$$(Cu/Q)_{\max} = e^{[a + b \ln H + c (\ln H)^2 + d (\ln H)^3]}$$

$H$  = effective stack height, stack height + plume rise, m

**Values of Curve-Fit Constants for Estimating  $(Cu/Q)_{\max}$  from  $H$  as a Function of Atmospheric Stability**

Stability	Constants			
	$a$	$b$	$c$	$d$
A	-1.0563	-2.7153	0.1261	0
B	-1.8060	-2.1912	0.0389	0
C	-1.9748	-1.9980	0	0
D	-2.5302	-1.5610	-0.0934	0
E	-1.4496	-2.5910	0.2181	-0.0343
F	-1.0488	-3.2252	0.4977	-0.0765

Adapted from Ranchoux, R.J.P., 1976.

♦ Turner, D.B., "Workbook of Atmospheric Dispersion Estimates: An Introduction to Dispersion Modeling," 2nd ed., Lewis Publishing/CRC Press, Florida, 1994.

## Cyclone

### Cyclone Collection (Particle Removal) Efficiency

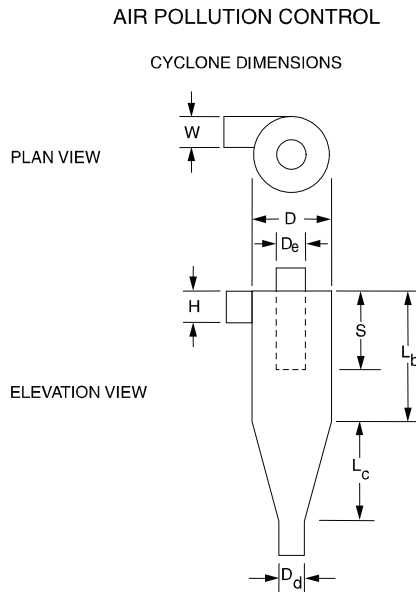
$$\eta = \frac{1}{1 + (d_{pc}/d_p)^2}, \text{ where}$$

$d_{pc}$  = diameter of particle collected with 50% efficiency

$d_p$  = diameter of particle of interest

$\eta$  = fractional particle collection efficiency

◆



### Cyclone 50% Collection Efficiency for Particle Diameter

$$d_{pc} = \left[ \frac{9\mu W}{2\pi N_e V_i (\rho_p - \rho_g)} \right]^{0.5}, \text{ where}$$

$d_{pc}$  = diameter of particle that is collected with 50% efficiency (m)

$\mu$  = dynamic viscosity of gas (kg/m•s)

$W$  = inlet width of cyclone (m)

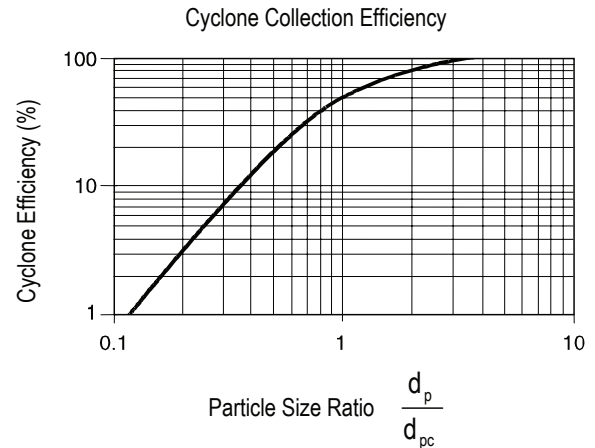
$N_e$  = number of effective turns gas makes in cyclone

$V_i$  = inlet velocity into cyclone (m/s)

$\rho_p$  = density of particle (kg/m<sup>3</sup>)

$\rho_g$  = density of gas (kg/m<sup>3</sup>)

◆



### Cyclone Effective Number of Turns Approximation

$$N_e = \frac{1}{H} \left[ L_b + \frac{L_c}{2} \right], \text{ where}$$

$N_e$  = number of effective turns gas makes in cyclone

$H$  = inlet height of cyclone (m)

$L_b$  = length of body cyclone (m)

$L_c$  = length of cone of cyclone (m)

◆

### Cyclone Ratio of Dimensions to Body Diameter

Dimension	High Efficiency	Conventional	High Throughput
Inlet height, $H$	0.44	0.50	0.80
Inlet width, $W$	0.21	0.25	0.35
Body length, $L_b$	1.40	1.75	1.70
Cone length, $L_c$	2.50	2.00	2.00
Vortex finder length, $S$	0.50	0.60	0.85
Gas exit diameter, $D_e$	0.40	0.50	0.75
Dust outlet diameter, $D_d$	0.40	0.40	0.40

◆ Adapted from Cooper, David C., and F.C. Alley, *Air Pollution Control: A Design Approach*, 2nd ed., Waveland Press, Illinois, 1986.

## Baghouse

### Air-to-Cloth Ratio for Baghouses

Dust	Shaker/Woven Reverse Air/Woven [m <sup>3</sup> /(min•m <sup>2</sup> )]	Pulse Jet/Felt [m <sup>3</sup> /(min•m <sup>2</sup> )]
alumina	0.8	2.4
asbestos	0.9	3.0
bauxite	0.8	2.4
carbon black	0.5	1.5
coal	0.8	2.4
cocoa	0.8	3.7
clay	0.8	2.7
cement	0.6	2.4
cosmetics	0.5	3.0
enamel frit	0.8	2.7
feeds, grain	1.1	4.3
feldspar	0.7	2.7
fertilizer	0.9	2.4
flour	0.9	3.7
fly ash	0.8	1.5
graphite	0.6	1.5
gypsum	0.6	3.0
iron ore	0.9	3.4
iron oxide	0.8	2.1
iron sulfate	0.6	1.8
lead oxide	0.6	1.8
leather dust	1.1	3.7
lime	0.8	3.0
limestone	0.8	2.4
mica	0.8	2.7
paint pigments	0.8	2.1
paper	1.1	3.0
plastics	0.8	2.1
quartz	0.9	2.7
rock dust	0.9	2.7
sand	0.8	3.0
sawdust (wood)	1.1	3.7
silica	0.8	2.1
slate	1.1	3.7
soap detergents	0.6	1.5
spices	0.8	3.0
starch	0.9	2.4
sugar	0.6	2.1
talc	0.8	3.0
tobacco	1.1	4.0

U.S. EPA OAQPS Control Cost Manual, 4th ed., EPA 450/3-90-006 (NTIS PB 90-169954), January 1990.

## Electrostatic Precipitator Efficiency

Deutsch-Anderson equation:

$$\eta = 1 - e^{(-WA/Q)}$$

where

$\eta$  = fractional collection efficiency

$W$  = terminal drift velocity

$A$  = total collection area

$Q$  = volumetric gas flow rate

Note that any consistent set of units can be used for  $W$ ,  $A$ , and  $Q$  (for example, ft/min, ft<sup>2</sup>, and ft<sup>3</sup>/min).

## Incineration

$$DRE = \frac{W_{in} - W_{out}}{W_{in}} \times 100\%$$

where

$DRE$  = destruction and removal efficiency (%)

$W_{in}$  = mass feed rate of a particular POHC (kg/h or lb/h)

$W_{out}$  = mass emission rate of the same POHC (kg/h or lb/h)

POHC = principal organic hazardous contaminant

$$CE = \frac{CO_2}{CO_2 + CO} \times 100\%$$

$CO_2$  = volume concentration (dry) of  $CO_2$   
(parts per million, volume, ppm<sub>v</sub>)

$CO$  = volume concentration (dry) of  $CO$  (ppm<sub>v</sub>)

$CE$  = combustion efficiency

## Kiln Formula

$$t = \frac{2.28 L/D}{SN}$$

where

$t$  = mean residence time, min

$L/D$  = internal length-to-diameter ratio

$S$  = kiln rake slope, in./ft of length

$N$  = rotational speed, rev/min

## Energy Content of Waste

Typical Waste Values	Moisture, %	Energy, Btu/lb
Food Waste	70	2,000
Paper	6	7,200
Cardboard	5	7,000
Plastics	2	14,000
Wood	20	8,000
Glass	2	60
Bi-metallic Cans	3	300

## FATE AND TRANSPORT

### Mass Calculations

Mass balance:  $\text{Mass}_{\text{in}} = \text{Mass}_{\text{out}}$

$M = CQ = CV$

Continuity equation =  $Q = vA$

$M$  = mass

$C$  = concentration

$Q$  = flow rate

$V$  = volume

$v$  = velocity

$A$  = cross-sectional area of flow

$M$  (lb/day) =  $C$  (mg/L)  $\times Q$  (MGD)  $\times 8.34$  [lb-L/(mg-MG)]

where:

MGD = million gallons per day

MG = million gallons

### Microbial Kinetics

#### BOD Exertion

$$y_t = L(1 - e^{-kt})$$

where

$k$  = BOD decay rate constant (base e, days<sup>-1</sup>)

$L$  = ultimate BOD (mg/L)

$t$  = time (days)

$y_t$  = the amount of BOD exerted at time  $t$  (mg/L)

#### Stream Modeling

Streeter Phelps

$$D = \frac{k_1 L_0}{k_2 - k_1} [\exp(-k_1 t) - \exp(-k_2 t)] + D_0 \exp(-k_2 t)$$

$$t_c = \frac{1}{k_2 - k_1} \ln \left[ \frac{k_2}{k_1} \left( 1 - D_0 \frac{(k_2 - k_1)}{k_1 L_0} \right) \right]$$

$$DO = DO_{\text{sat}} - D$$

where

$D$  = dissolved oxygen deficit (mg/L)

$DO$  = dissolved oxygen concentration (mg/L)

$D_0$  = initial dissolved oxygen deficit in mixing zone (mg/L)

$DO_{\text{sat}}$  = saturated dissolved oxygen concentration (mg/L)

$k_1$  = deoxygenation rate constant, base e (days<sup>-1</sup>)

$k_2$  = reaeration rate constant, base e (days<sup>-1</sup>)

$L_0$  = initial BOD ultimate in mixing zone (mg/L)

$t$  = time (days)

$t_c$  = time at which minimum dissolved oxygen occurs (days)

### Monod Kinetics—Substrate Limited Growth

Continuous flow systems where growth is limited by one substrate (chemostat):

$$\mu = \frac{Y k_m S}{K_s + S} - k_d = \mu_{\text{max}} \frac{S}{K_s + S} - k_d$$

### Multiple Limiting Substrates

$$\frac{\mu}{\mu_{\text{max}}} = [\mu_1(S_1)][\mu_2(S_2)][\mu_3(S_3)] \dots [\mu_n(S_n)]$$

$$\text{where } \mu_i = \frac{S_i}{K_{s_i} + S_i} \text{ for } i = 1 \text{ to } n$$

### Non-steady State Continuous Flow

$$\frac{dx}{dt} = Dx_0 + (\mu - k_d - D)x$$

### Steady State Continuous Flow

$$\mu = D \text{ with } k_d \ll \mu$$

### Product production at steady state, single substrate limiting

$$X_1 = Y_{P/S}(S_0 - S_i)$$

$k_d$  = microbial death rate or endogenous decay rate constant (time<sup>-1</sup>)

$k_m$  = maximum growth rate constant (time<sup>-1</sup>)

$K_s$  = saturation constant or half-velocity constant  
[= concentration at  $\mu_{\text{max}}/2$ ]

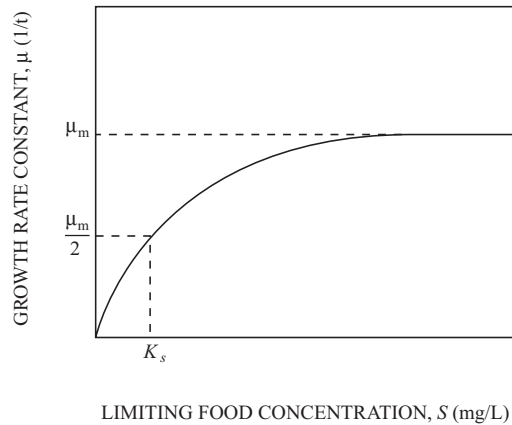
$S$  = concentration of substrate in solution (mass/unit volume)

$Y$  = yield coefficient [(mass/L product)/(mass/L food used)]

$\mu$  = specific growth rate (time<sup>-1</sup>)

$\mu_{\text{max}}$  = maximum specific growth rate (time<sup>-1</sup>) =  $Yk_m$

- ♦ Monod growth rate constant as a function of limiting food concentration.



- $X_1$  = product (mg/L)
- $V_r$  = volume (L)
- $D$  = dilution rate (flow  $f$ /reactor volume  $V_r$ ;  $\text{hr}^{-1}$ )
- $f$  = flow rate (L/hr)
- $\mu_i$  = growth rate with one or multiple limiting substrates ( $\text{hr}^{-1}$ )
- $S_i$  = substrate  $i$  concentration (mass/unit volume)
- $S_0$  = initial substrate concentration (mass/unit volume)
- $Y_{P/S}$  = product yield per unit of substrate (mass/mass)
- $p$  = product concentration (mass/unit volume)
- $x$  = cell concentration (mass/unit volume)
- $x_0$  = initial cell concentration (mass/unit volume)
- $t$  = time (time)

#### Kinetic Temperature Corrections

$$k_T = k_{20} (\theta)^{T-20}$$

Activated sludge:  $\theta = 1.136$  ( $T > 20^\circ\text{C}$ )  
 $\theta = 1.056$  ( $T < 20^\circ\text{C}$ )

Reaeration  $\theta = 1.024$

Biotowers  $\theta = 1.035$

Trickling Filters  $\theta = 1.072$

- ♦ Davis, M.L., and D. Cornwell, *Introduction to Environmental Engineering*, 3rd ed., McGraw-Hill, 1998.

#### **Partition Coefficients**

##### Bioconcentration Factor $BCF$

The amount of a chemical to accumulate in aquatic organisms.

$$BCF = C_{\text{org}}/C$$

where

$C_{\text{org}}$  = equilibrium concentration in organism (mg/kg or ppm)

$C$  = concentration in water (ppm)

##### Octanol-Water Partition Coefficient

The ratio of a chemical's concentration in the octanol phase to its concentration in the aqueous phase of a two-phase octanol-water system.

$$K_{ow} = C_o/C_w$$

where

$C_o$  = concentration of chemical in octanol phase (mg/L or  $\mu\text{g/L}$ )

$C_w$  = concentration of chemical in aqueous phase (mg/L or  $\mu\text{g/L}$ )

##### Organic Carbon Partition Coefficient $K_{oc}$

$$K_{oc} = C_{\text{soil}}/C_{\text{water}}$$

where

$C_{\text{soil}}$  = concentration of chemical in organic carbon component of soil ( $\mu\text{g}$  adsorbed/kg organic C, or ppb)

$C_{\text{water}}$  = concentration of chemical in water (ppb or  $\mu\text{g/kg}$ )

##### Retardation Factor $R$

$$R = 1 + (\rho/\eta)K_d$$

where

$\rho$  = bulk density

$\eta$  = porosity

$K_d$  = distribution coefficient

##### Soil - Water Partition Coefficient $K_{sw} = K_p$

$$K_{sw} = X/C$$

where

$X$  = concentration of chemical in soil (ppb or  $\mu\text{g/kg}$ )

$C$  = concentration of chemical in water (ppb or  $\mu\text{g/kg}$ )

$$K_{sw} = K_{oc}f_{oc}$$

$f_{oc}$  = fraction of organic carbon in the soil (dimensionless)



♦ **Steady-State Reactor Parameters (Constant Density Systems)**

**Comparison of Steady-State Retention Times ( $\theta$ ) for Decay Reactions of Different Order<sup>a</sup>**

Reaction Order	r	Equations for Mean Retention Times ( $\theta$ )		
		Ideal Batch	Ideal Plug Flow	Ideal CMFR
Zero <sup>b</sup>	$-k$	$\frac{(C_o - C_t)}{k}$	$\frac{(C_o - C_t)}{k}$	$\frac{(C_o - C_t)}{k}$
First	$-kC$	$\frac{\ln(C_o/C_t)}{k}$	$\frac{\ln(C_o/C_t)}{k}$	$\frac{(C_o/C_t) - 1}{k}$
Second	$-kC^2$	$\frac{(C_o/C_t) - 1}{kC_o}$	$\frac{(C_o/C_t) - 1}{kC_o}$	$\frac{(C_o/C_t) - 1}{kC_t}$

<sup>a</sup> $C_o$  = initial concentration or influent concentration;  $C_t$  = final condition or effluent concentration.

<sup>b</sup>Expressions are valid for  $k\theta \leq C_o$ ; otherwise  $C_t = 0$ .

**Comparison of Steady-State Performance for Decay Reactions of Different Order<sup>a</sup>**

Reaction Order	r	Equations for $C_t$		
		Ideal Batch	Ideal Plug Flow	Ideal CMFR
Zero <sup>b</sup> $t \leq C_o/k$	$-k$	$C_o - kt$	$C_o - k\theta$	$C_o - k\theta$
$t > C_o/k$		0		
First	$-kC$	$C_o[\exp(-kt)]$	$C_o[\exp(-k\theta)]$	$\frac{C_o}{1 + k\theta}$
Second	$-kC^2$	$\frac{C_o}{1 + ktC_o}$	$\frac{C_o}{1 + k\theta C_o}$	$\frac{(4k\theta C_o + 1)^{1/2} - 1}{2k\theta}$

<sup>a</sup> $C_o$  = initial concentration or influent concentration;  $C_t$  = final condition or effluent concentration.

<sup>b</sup>Time conditions are for ideal batch reactor only.

♦ Davis, M.L., and S.J. Masten, *Principles of Environmental Engineering and Science*, 2nd ed., McGraw-Hill, 2004.

## LANDFILL

### Break-Through Time for Leachate to Penetrate a Clay Liner

$$t = \frac{d^2 \eta}{K(d + h)}$$

where

$t$  = breakthrough time (yr)

$d$  = thickness of clay liner (ft)

$\eta$  = porosity

$K$  = hydraulic conductivity (ft/yr)

$h$  = hydraulic head (ft)

Typical porosity values for clays with a coefficient of permeability in the range of  $10^{-6}$  to  $10^{-8}$  cm/s vary from 0.1 to 0.3.

### Effect of Overburden Pressure

$$SW_p = SW_i + \frac{p}{a + bp}$$

where

$SW_p$  = specific weight of the waste material at pressure  $p$  (lb/yd<sup>3</sup>) (typical 1,750 to 2,150)

$SW_i$  = initial compacted specific weight of waste (lb/yd<sup>3</sup>) (typical 1,000)

$p$  = overburden pressure (lb/in<sup>2</sup>)

$a$  = empirical constant (yd<sup>3</sup>/in<sup>2</sup>)

$b$  = empirical constant (yd<sup>3</sup>/lb)

### Gas Flux

$$N_A = \frac{D\eta^{4/3}(C_{A_{\text{atm}}} - C_{A_{\text{fill}}})}{L}$$

where

$N_A$  = gas flux of compound  $A$ , [g/(cm<sup>2</sup> • s)][lb • mol/(ft<sup>2</sup> • d)]

$C_{A_{\text{atm}}}$  = concentration of compound  $A$  at the surface of the landfill cover, g/cm<sup>3</sup> (lb • mol/ft<sup>3</sup>)

$C_{A_{\text{fill}}}$  = concentration of compound  $A$  at the bottom of the landfill cover, g/cm<sup>3</sup> (lb • mol/ft<sup>3</sup>)

$L$  = depth of the landfill cover, cm (ft)

Typical values for the coefficient of diffusion for methane and carbon dioxide are 0.20 cm<sup>2</sup>/s (18.6 ft<sup>2</sup>/d) and 0.13 cm<sup>2</sup>/s (12.1 ft<sup>2</sup>/d), respectively.

$D$  = diffusion coefficient, cm<sup>2</sup>/s (ft<sup>2</sup>/d)

$\eta_{\text{gas}}$  = gas-filled porosity, cm<sup>3</sup>/cm<sup>3</sup> (ft<sup>3</sup>/ft<sup>3</sup>)

$\eta$  = porosity, cm<sup>3</sup>/cm<sup>3</sup> (ft<sup>3</sup>/ft<sup>3</sup>)

### Soil Landfill Cover Water Balance

$$\Delta S_{\text{LC}} = P - R - \text{ET} - \text{PER}_{\text{sw}}$$

where

$\Delta S_{\text{LC}}$  = change in the amount of water held in storage in a unit volume of landfill cover (in.)

$P$  = amount of precipitation per unit area (in.)

$R$  = amount of runoff per unit area (in.)

$\text{ET}$  = amount of water lost through evapotranspiration per unit area (in.)

$\text{PER}_{\text{sw}}$  = amount of water percolating through the unit area of landfill cover into compacted solid waste (in.)

## POPULATION MODELING

### Population Projection Equations

#### Linear Projection = Algebraic Projection

$$P_t = P_0 + k\Delta t$$

where

$P_t$  = population at time  $t$

$P_0$  = population at time zero

$k$  = growth rate

$\Delta t$  = elapsed time in years relative to time zero

#### Log Growth = Exponential Growth = Geometric Growth

$$P_t = P_0 e^{k\Delta t}$$

$$\ln P_t = \ln P_0 + k\Delta t$$

where

$P_t$  = population at time  $t$

$P_0$  = population at time zero

$k$  = growth rate

$\Delta t$  = elapsed time in years relative to time zero

#### Percent Growth

$$P_t = P_0(1 + k)^n$$

where

$P_t$  = population at time  $t$

$P_0$  = population at time zero

$k$  = growth rate

$n$  = number of periods

#### Ratio and Correlation Growth

$$\frac{P_2}{P_{2R}} = \frac{P_1}{P_{1R}} = k$$

where

$P_2$  = projected population

$P_{2R}$  = projected population of a larger region

$P_1$  = population at last census

$P_{1R}$  = population of larger region at last census

$k$  = growth ratio constant

#### Decreasing-Rate-of-Increase Growth

$$P_t = P_0 + (S - P_0)(1 - e^{-k(t-t_0)})$$

where

$P_t$  = population at time  $t$

$P_0$  = population at time zero

$k$  = growth rate constant

$S$  = saturation population

$t, t_0$  = future time, initial time

## RADIATION

### Effective Half-Life

Effective half-life,  $\tau_e$ , is the combined radioactive and biological half-life.

$$\frac{1}{\tau_e} = \frac{1}{\tau_r} + \frac{1}{\tau_b}$$

where

$\tau_r$  = radioactive half-life

$\tau_b$  = biological half-life

### Half-Life

$$N = N_0 e^{-0.693 t/\tau}$$

where

$N_0$  = original number of atoms

$N$  = final number of atoms

$t$  = time

$\tau$  = half-life

Flux at distance 2 = (Flux at distance 1)  $(r_1/r_2)^2$

where  $r_1$  and  $r_2$  are distances from source.

The half-life of a biologically degraded contaminant assuming a first-order rate constant is given by:

$$t_{1/2} = \frac{0.693}{k}$$

$k$  = rate constant ( $\text{time}^{-1}$ )

$t_{1/2}$  = half-life (time)

### Daughter Product Activity

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

where  $\lambda_{1,2}$  = decay constants ( $\text{time}^{-1}$ )

$N_{10}$  = initial activity (curies) of parent nuclei

$t$  = time

### Daughter Product Maximum Activity Time

$$t' = \frac{\ln \lambda_2 - \ln \lambda_1}{\lambda_2 - \lambda_1}$$

### Inverse Square Law

$$\frac{I_1}{I_2} = \frac{(R_2)^2}{(R_1)^2}$$

where  $I_{1,2}$  = Radiation intensity at locations 1 and 2

$R_{1,2}$  = Distance from the source at locations 1 and 2

## SAMPLING AND MONITORING

### Data Quality Objectives (DQO) for Sampling Soils and Solids

Investigation Type	Confidence Level (1- $\alpha$ ) (%)	Power (1- $\beta$ ) (%)	Minimum Detectable Relative Difference (%)
Preliminary site investigation	70–80	90–95	10–30
Emergency clean-up	80–90	90–95	10–20
Planned removal and remedial response operations	90–95	90–95	10–20

Confidence level:  $1 - (\text{Probability of a Type I error}) = 1 - \alpha$  = size probability of not making a Type I error.

Power =  $1 - (\text{Probability of a Type II error}) = 1 - \beta$  = probability of not making a Type II error.

EPA Document "EPA/600/8-89/046" *Soil Sampling Quality Assurance User's Guide*, Chapter 7.

$$CV = (100 * s) / \bar{x}$$

CV = coefficient of variation

$s$  = standard deviation of sample

$\bar{x}$  = sample average

Minimum Detectable Relative Difference = Relative increase over background  $[100 (\mu_s - \mu_B) / \mu_B]$  to be detectable with a probability  $(1 - \beta)$

**Number of Samples Required in a One-Sided One-Sample t-Test to Achieve a Minimum Detectable Relative Difference at Confidence Level (1- $\alpha$ ) and Power (1- $\beta$ )**

Coefficient of Variation (%)	Power (%)	Confidence Level (%)	Minimum Detectable Relative Difference (%)				
			5	10	20	30	40
15	95	99	145	39	12	7	5
		95	99	26	8	5	3
		90	78	21	6	3	3
		80	57	15	4	2	2
	90	99	120	32	11	6	5
		95	79	21	7	4	3
		90	60	16	5	3	2
		80	41	11	3	2	1
	80	99	94	26	9	6	5
		95	58	16	5	3	3
		90	42	11	4	2	2
		80	26	7	2	2	1
25	95	99	397	102	28	14	9
		95	272	69	19	9	6
		90	216	55	15	7	5
		80	155	40	11	5	3
	90	99	329	85	24	12	8
		95	272	70	19	9	6
		90	166	42	12	6	4
		80	114	29	8	4	3
	80	99	254	66	19	10	7
		95	156	41	12	6	4
		90	114	30	8	4	3
		80	72	19	5	3	2
35	95	99	775	196	42	25	15
		95	532	134	35	17	10
		90	421	106	28	13	8
		80	304	77	20	9	6
	90	99	641	163	43	21	13
		95	421	107	28	14	8
		90	323	82	21	10	6
		80	222	56	15	7	4
	80	99	495	126	34	17	11
		95	305	78	21	10	7
		90	222	57	15	7	5
		80	140	36	10	5	3

## WASTEWATER TREATMENT AND TECHNOLOGIES

### Activated Sludge

$$X_A = \frac{\theta_c Y (S_0 - S_e)}{\theta (1 + k_d \theta_c)}$$

Steady State Mass Balance around Secondary Clarifier:

$$(Q_0 + Q_R)X_A = Q_e X_e + Q_R X_r + Q_w X_w$$

$$\theta_c = \text{Solids residence time} = \frac{V(X_A)}{Q_w X_w + Q_e X_e}$$

$$\text{Sludge volume/day: } Q_s = \frac{M(100)}{\rho_s (\% \text{ solids})}$$

$$\text{SVI} = \frac{\text{Sludge volume after settling (mL/L)} * 1,000}{\text{MLSS (mg/L)}}$$

$k_d$  = microbial death ratio; kinetic constant; day<sup>-1</sup>; typical range 0.1–0.01, typical domestic wastewater value = 0.05 day<sup>-1</sup>

$S_e$  = effluent BOD or COD concentration (kg/m<sup>3</sup>)

$S_0$  = influent BOD or COD concentration (kg/m<sup>3</sup>)

$X_A$  = biomass concentration in aeration tank (MLSS or MLVSS kg/m<sup>3</sup>)

$Y$  = yield coefficient (kg biomass/kg BOD or COD consumed); range 0.4–1.2

$\theta$  = hydraulic residence time =  $V/Q$

◆

### Design and Operational Parameters for Activated-Sludge Treatment of Municipal Wastewater

Type of Process	Mean cell residence time ( $\theta_c$ , d)	Food-to-mass ratio [(kg BOD <sub>5</sub> / (day·kg MLSS)]	Volumetric loading (kg BOD <sub>5</sub> /m <sup>3</sup> )	Hydraulic residence time in aeration basin ( $\theta$ , h)	Mixed liquor suspended solids (MLSS, mg/L)	Recycle ratio ( $Q_r/Q$ )	Flow regime*	BOD <sub>5</sub> removal efficiency (%)	Air supplied (m <sup>3</sup> /kg BOD <sub>5</sub> )
Tapered aeration	5–15	0.2–0.4	0.3–0.6	4–8	1,500–3,000	0.25–0.5	PF	85–95	45–90
Conventional	4–15	0.2–0.4	0.3–0.6	4–8	1,500–3,000	0.25–0.5	PF	85–95	45–90
Step aeration	4–15	0.2–0.4	0.6–1.0	3–5	2,000–3,500	0.25–0.75	PF	85–95	45–90
Completely mixed	4–15	0.2–0.4	0.8–2.0	3–5	3,000–6,000	0.25–1.0	CM	85–95	45–90
Contact stabilization	4–15	0.2–0.6	1.0–1.2			0.25–1.0			45–90
Contact basin				0.5–1.0	1,000–3,000		PF	80–90	
Stabilization basin				4–6	4,000–10,000		PF		
High-rate aeration	4–15	0.4–1.5	1.6–16	0.5–2.0	4,000–10,000	1.0–5.0	CM	75–90	25–45
Pure oxygen	8–20	0.2–1.0	1.6–4	1–3	6,000–8,000	0.25–0.5	CM	85–95	
Extended aeration	20–30	0.05–0.15	0.16–0.40	18–24	3,000–6,000	0.75–1.50	CM	75–90	90–125

\*PF = plug flow, CM = completely mixed.

### ◆ Blowers

$$P_w = \frac{W R T_1}{C n e} \left[ \left( \frac{P_2}{P_1} \right)^{0.283} - 1 \right]$$

$C$  = 29.7 (constant for SI unit conversion)  
= 550 ft·lb/(sec·hp) (U.S. Customary Units)

$P_w$  = power requirement (hp)

$W$  = weight of flow of air (lb/sec)

$R$  = engineering gas constant for air = 53.3 ft·lb/(lb air·°R)

Solids loading rate =  $Q X_A$

For activated sludge secondary clarifier  $Q = Q_0 + Q_R$

Organic loading rate (volumetric) =  $Q_0 S_0 / Vol$

Organic loading rate (F:M) =  $Q_0 S_0 / (Vol X_A)$

Organic loading rate (surface area) =  $Q_0 S_0 / A_M$

$\rho_s$  = density of solids

$A$  = surface area of unit

$A_M$  = surface area of media in fixed-film reactor

$A_x$  = cross-sectional area of channel

$M$  = sludge production rate (dry weight basis)

$Q_0$  = influent flow rate

$Q_e$  = effluent flow rate

$Q_w$  = waste sludge flow rate

$\rho_s$  = wet sludge density

$R$  = recycle ratio =  $Q_R/Q_0$

$Q_R$  = recycle flow rate =  $Q_0 R$

$X_e$  = effluent suspended solids concentration

$X_w$  = waste sludge suspended solids concentration

$V$  = aeration basin volume

$Q$  = flow rate

$X_r$  = recycled sludge suspended solids concentration

$T_1$  = absolute inlet temperature (°R)

$P_1$  = absolute inlet pressure (lbf/in<sup>2</sup>)

$P_2$  = absolute outlet pressure (lbf/in<sup>2</sup>)

$n$  =  $(k - 1)/k = 0.283$  for air

$e$  = efficiency (usually 0.70 <  $e$  < 0.90)

◆ Metcalf and Eddy, *Wastewater Engineering: Treatment, Disposal, and Reuse*, 3rd ed., McGraw-Hill, 1991.

## Facultative Pond

BOD Loading Total System  $\leq 35$  pounds BOD<sub>5</sub>/(acre-day)

Minimum = 3 ponds

Depth = 3–8 ft

Minimum  $t = 90$ –120 days

## Biotower

### Fixed-Film Equation without Recycle

$$\frac{S_e}{S_0} = e^{-kD/q^n}$$

### Fixed-Film Equation with Recycle

$$\frac{S_e}{S_a} = \frac{e^{-kD/q^n}}{(1 + R) - R(e^{-kD/q^n})}$$

where

$S_e$  = effluent BOD<sub>5</sub> (mg/L)

$S_0$  = influent BOD<sub>5</sub> (mg/L)

$R$  = recycle ratio =  $Q_0/Q_R$

$Q_R$  = recycle flow rate

$$S_a = \frac{S_0 + RS_e}{1 + R}$$

$D$  = depth of biotower media (m)

$q$  = hydraulic loading [ $\text{m}^3/(\text{m}^2 \cdot \text{min})$ ]

$$= (Q_0 + RQ_0)/A_{\text{plan}} \text{ (with recycle)}$$

$k$  = treatability constant; functions of wastewater and medium ( $\text{min}^{-1}$ ); range 0.01–0.1; for municipal wastewater and modular plastic media  $0.06 \text{ min}^{-1}$  @  $20^\circ\text{C}$

$$k_T = k_{20}(1.035)^{T-20}$$

$n$  = coefficient relating to media characteristics; modular plastic,  $n = 0.5$

## ♦ Aerobic Digestion

### Design criteria for aerobic digesters<sup>a</sup>

Parameter	Value
Sludge retention time, d	
At $20^\circ\text{C}$	40
At $15^\circ\text{C}$	60
Solids loading, lb volatile solids/ $\text{ft}^3 \cdot \text{d}$	0.1–0.3
Oxygen requirements, lb O <sub>2</sub> /lb solids destroyed	
Cell tissue	~2.3
BOD <sub>5</sub> in primary sludge	1.6–1.9
Energy requirements for mixing	
Mechanical aerators, hp/ $10^3 \text{ ft}^3$	0.7–1.50
Diffused-air mixing, $\text{ft}^3/10^3 \text{ ft}^3 \cdot \text{min}$	20–40
Dissolved-oxygen residual in liquid, mg/L	1–2
Reduction in volatile suspended solids, %	40–50

### Tank Volume

$$V = \frac{Q_i(X_i + FS_i)}{X_d(k_d P_v + 1/\theta_c)}$$

where

$V$  = volume of aerobic digester ( $\text{ft}^3$ )

$Q_i$  = influent average flowrate to digester ( $\text{ft}^3/\text{d}$ )

$X_i$  = influent suspended solids (mg/L)

$F$  = fraction of the influent BOD<sub>5</sub> consisting of raw primary sludge (expressed as a decimal)

$S_i$  = influent BOD<sub>5</sub> (mg/L)

$X_d$  = digester suspended solids (mg/L); typically  $X_d = (0.7)X_i$

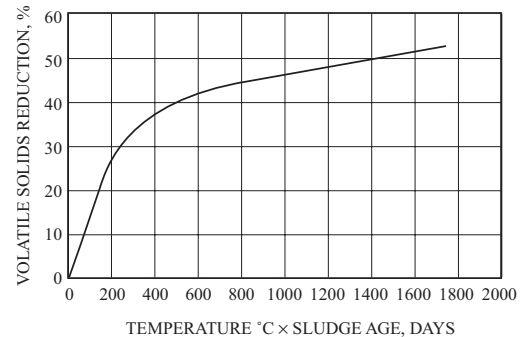
$k_d$  = reaction-rate constant ( $\text{d}^{-1}$ )

$P_v$  = volatile fraction of digester suspended solids (expressed as a decimal)

$\theta_c$  = solids residence time (sludge age) (d)

$FS_i$  can be neglected if primary sludge is not included on the sludge flow to the digester.

♦



VOLATILE SOLIDS REDUCTION IN AN AEROBIC DIGESTER AS A FUNCTION OF DIGESTER LIQUID TEMPERATURE AND DIGESTER SLUDGE AGE

## • Anaerobic Digestion

### Design parameters for anaerobic digesters

Parameter	Standard-rate	High-rate
Solids residence time, d	30–90	10–20
Volatile solids loading, $\text{kg}/\text{m}^3/\text{d}$	0.5–1.6	1.6–6.4
Digested solids concentration, %	4–6	4–6
Volatile solids reduction, %	35–50	45–55
Gas production ( $\text{m}^3/\text{kg}$ VSS added)	0.5–0.55	0.6–0.65
Methane content, %	65	65

### Standard Rate

$$\text{Reactor Volume} = \frac{V_1 + V_2}{2} t_r + V_2 t_s$$

### High Rate

#### First stage

$$\text{Reactor Volume} = V_1 t_r$$

#### Second Stage

$$\text{Reactor Volume} = \frac{V_1 + V_2}{2} t_t + V_2 t_s$$

where

$V_1$  = raw sludge input (volume/day)

$V_2$  = digested sludge accumulation (volume/day)

$t_r$  = time to react in a high-rate digester = time to react and thicken in a standard-rate digester

$t_t$  = time to thicken in a high-rate digester

$t_s$  = storage time

♦ Tchobanoglous, G., and Metcalf and Eddy, *Wastewater Engineering: Treatment, Disposal, and Reuse*, 4th ed., McGraw-Hill, 2003.

• Peavy, HS, D.R. Rowe, and G. Tchobanoglous, *Environmental Engineering*, McGraw-Hill, 1985.

## WATER TREATMENT TECHNOLOGIES

### Activated Carbon Adsorption

#### Freundlich Isotherm

$$\frac{x}{m} = X = KC_e^{1/n}$$

where

$x$  = mass of solute adsorbed

$m$  = mass of adsorbent

$X$  = mass ratio of the solid phase—that is, the mass of adsorbed solute per mass of adsorbent

$C_e$  = equilibrium concentration of solute, mass/volume

$K, n$  = experimental constants

Linearized Form

$$\ln \frac{x}{m} = \frac{1}{n} \ln C_e + \ln K$$

For linear isotherm,  $n = 1$

#### Langmuir Isotherm

$$\frac{x}{m} = X = \frac{aKC_e}{1 + KC_e}$$

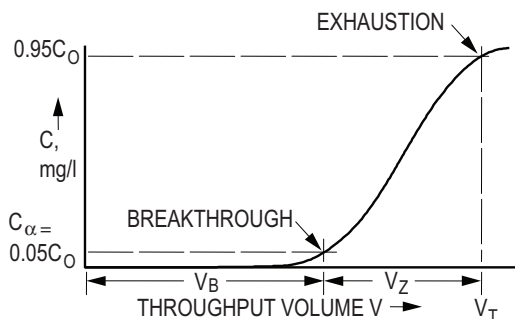
where

$a$  = mass of adsorbed solute required to saturate completely a unit mass of adsorbent

$K$  = experimental constant

Linearized Form

$$\frac{m}{x} = \frac{1}{a} + \frac{1}{aK} \frac{1}{C_e}$$



#### Depth of Sorption Zone

$$Z_s = Z \left[ \frac{V_Z}{V_T - 0.5V_Z} \right]$$

where

$$V_Z = V_T - V_B$$

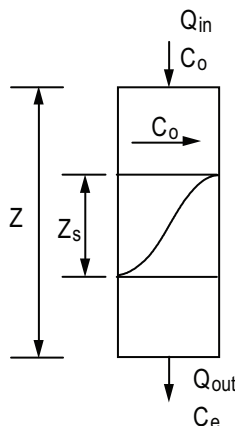
$Z_s$  = depth of sorption zone

$Z$  = total carbon depth

$V_T$  = total volume treated at exhaustion ( $C = 0.95 C_0$ )

$V_B$  = total volume at breakthrough ( $C = C_\alpha = 0.05 C_0$ )

$C_0$  = concentration of contaminant in influent



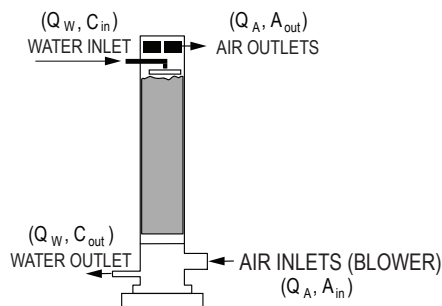
### Air Stripping

$P_i = HC_i$  = Henry's Law

$P_i$  = partial pressure of component  $i$ , atm

$H$  = Henry's Law constant, atm·m<sup>3</sup>/kmol

$C_i$  = concentration of component  $i$  in solvent, kmol/m<sup>3</sup>



$$A_{out} = HC_{in}$$

$$Q_W \cdot C_{in} = Q_A H C_{in}$$

$$Q_W = Q_A H$$

$$H' (Q_A / Q_W) = 1$$

where

$A_{out}$  = concentration in the effluent air (kmol/m<sup>3</sup>); in this formulation of the equation  $A_{in}$  and  $C_{out}$  are assumed to be negligible for simplicity.

$Q_W$  = water flow rate (m<sup>3</sup>/s)

$Q_A$  = air flow rate (m<sup>3</sup>/s)

$A_{in}$  = concentration of contaminant in air (kmol/m<sup>3</sup>)

$C_{out}$  = concentration of contaminants in effluent water (kmol/m<sup>3</sup>)

$C_{in}$  = concentration of contaminants in influent water (kmol/m<sup>3</sup>)

#### Stripper Packing Height = $Z$

$$Z = HTU \times NTU$$

Assuming rapid equilibrium:

$$NTU = \left( \frac{R_S}{R_S - 1} \right) \ln \left( \frac{(C_{in}/C_{out})(R_S - 1) + 1}{R_S} \right)$$

where

$NTU$  = number of transfer units

$H$  = Henry's Law constant

$H' = H/RT$  = dimensionless Henry's Law constant

$T$  = temperature in units consistent with  $R$

$R$  = universal gas constant, atm·m<sup>3</sup>/(kmol·K)

$R_S$  = stripping factor  $H'(Q_A/Q_W)$

$C_{in}$  = concentration in the influent water (kmol/m<sup>3</sup>)

$C_{out}$  = concentration in the effluent water (kmol/m<sup>3</sup>)

$HTU$  = Height of Transfer Units =  $\frac{L}{M_W K_L a}$ , where

$L$  = liquid molar loading rate [kmol/(s·m<sup>2</sup>)]

$M_W$  = molar density of water  
(55.6 kmol/m<sup>3</sup>) = 3.47 lbmol/ft<sup>3</sup>

$K_L a$  = overall transfer rate constant (s<sup>-1</sup>)

## Clarifier

Overflow rate = Hydraulic loading rate =  $v_o = Q/A_{\text{surface}}$

$v_o$  = critical settling velocity

= terminal settling velocity of smallest particle that is 100% removed

Weir loading = weir overflow rate, WOR =  $Q/\text{Weir Length}$

Horizontal velocity = approach velocity =  $v_h$

$$= Q/A_{\text{cross-section}} = Q/A_x$$

Hydraulic residence time =  $V/Q = \theta$

where

$Q$  = flow rate

$A_x$  = cross-sectional area

$A$  = surface area, plan view

$V$  = tank volume

### Typical Primary Clarifier Efficiency Percent Removal

	Overflow rates			
	1,200 (gpd/ft <sup>2</sup> )	1,000 (gpd/ft <sup>2</sup> )	800 (gpd/ft <sup>2</sup> )	600 (gpd/ft <sup>2</sup> )
	48.9 (m/d)	40.7 (m/d)	32.6 (m/d)	24.4 (m/d)
	54%	58%	64%	68%
Suspended Solids				
BOD <sub>5</sub>	30%	32%	34%	36%

## Weir Loadings

1. Water Treatment—weir overflow rates should not exceed 20,000 gpd/ft
2. Wastewater Treatment
  - a. Flow  $\leq$  1 MGD: weir overflow rates should not exceed 10,000 gpd/ft
  - b. Flow  $>$  1 MGD: weir overflow rates should not exceed 15,000 gpd/ft

## Horizontal Velocities

1. Water Treatment—horizontal velocities should not exceed 0.5 fpm
2. Wastewater Treatment—no specific requirements (use the same criteria as for water)

## Dimensions

1. Rectangular Tanks
  - a. Length:Width ratio = 3:1 to 5:1
  - b. Basin width is determined by the scraper width (or multiples of the scraper width)
  - c. Bottom slope is set at 1%
2. Circular Tanks
  - a. Diameters up to 200 ft
  - b. Diameters must match the dimensions of the sludge scraping mechanism
  - c. Bottom slope is less than 8%

## Design Criteria for Sedimentation Basins

Type of Basin	Overflow Rate				Solids Loading Rate				Hydraulic Residence Time (hr)	Depth (ft)
	Average (gpd/ft <sup>2</sup> )	Peak (m <sup>3</sup> /m <sup>2</sup> ·d)	Average (gpd/ft <sup>2</sup> )	Peak (m <sup>3</sup> /m <sup>2</sup> ·d)	Average (lb/ft <sup>2</sup> ·d)	Peak (kg/m <sup>2</sup> ·h)	Average (lb/ft <sup>2</sup> ·h)	Peak (kg/m <sup>2</sup> ·h)		
<b>Water Treatment</b>										
Clarification following coagulation and flocculation:										
Alum coagulation	350–550	14–22							4–8	12–16
Ferric coagulation	550–700	22–28							4–8	12–16
Upflow clarifiers										
Groundwater	1,500–2,200	61–90							1	
Surface water	1,000–1,500	41–61							4	
Clarification following lime-soda softening										
Conventional	550–1,000	22–41							2–4	
Upflow clarifiers										
Groundwater	1,000–2,500	41–102							1	
Surface water	1,000–1,800	41–73							4	
<b>Wastewater Treatment</b>										
Primary clarifiers	800–1,200	32–49	1,200–2,000	50–80					2	10–12
Settling basins following fixed film reactors	400–800	16–33							2	
Settling basins following air-activated sludge reactors										
All configurations EXCEPT extended aeration	400–700	16–28							2	12–15
Extended aeration	200–400	8–16	1,000–1,200	40–64	19–29	4–6	38	8	2	12–15
Settling basins following chemical flocculation reactors	800–1,200		600–800	24–32	5–24	1–5	34	7	2	



## Settling Equations

### General Spherical

$$v_t = \sqrt{\frac{4g(\rho_p - \rho_f)d}{3C_D\rho_f}}$$

where

$C_D$  = drag coefficient  
 =  $24/\text{Re}$  (Laminar;  $\text{Re} \leq 1.0$ )  
 =  $24/\text{Re} + 3/(\text{Re}^{1/2}) + 0.34$  (Transitional)  
 =  $0.4$  (Turbulent;  $\text{Re} \geq 10^4$ )

$\text{Re}$  = Reynolds number  $\frac{v_t \rho d}{\mu}$

$g$  = gravitational constant

$\rho_p$  and  $\rho_f$  = density of particle and fluid respectively

$d$  = diameter of sphere

$\mu$  = bulk viscosity of liquid = absolute viscosity

$v_t$  = terminal settling velocity

### Stokes' Law

$$v_t = \frac{g(\rho_p - \rho_f)d^2}{18\mu} = \frac{g\rho_f(S.G. - 1)d^2}{18\mu}$$

Approach velocity = horizontal velocity =  $Q/A_x$

Hydraulic loading rate =  $Q/A$

Hydraulic residence time =  $V/Q = \theta$

where

$Q$  = flow rate

$A_x$  = cross-sectional area

$A$  = surface area, plan view

$V$  = tank volume

$\rho_f$  = fluid mass density

$S.G.$  = specific gravity

## Filtration Equations

Filter bay length:width ratio = 1.2:1 to 1.5:1

Effective size =  $d_{10}$

Uniformity coefficient =  $d_{60}/d_{10}$

$d_x$  = diameter of particle class for which  $x\%$  of sample is less than (units meters or feet)

Filter equations can be used with any consistent set of units.

## Head Loss Through Clean Bed

### Rose Equation

Monosized Media

$$h_f = \frac{1067(v_s)^2 LC_D}{g\eta^4 d}$$

Multisized Media

$$h_f = \frac{1067(v_s)^2 L}{g\eta^4} \sum \frac{C_{Dij} x_{ij}}{d_{ij}}$$

### Carmen-Kozeny Equation

Monosized Media

$$h_f = \frac{f' L (1 - \eta) v_s^2}{\eta^3 g d}$$

Multisized Media

$$h_f = \frac{L (1 - \eta) v_s^2}{\eta^3 g} \sum \frac{f'_{ij} x_{ij}}{d_{ij}}$$

$$f' = \text{friction factor} = 150 \left( \frac{1 - \eta}{\text{Re}} \right) + 1.75$$

where

$h_f$  = head loss through the clean bed (m of  $\text{H}_2\text{O}$ )

$L$  = depth of filter media (m)

$\eta$  = porosity of bed = void volume/total volume

$v_s$  = filtration rate = empty bed approach velocity

=  $Q/A_{\text{plan}}$  (m/s)

$g$  = gravitational acceleration ( $\text{m/s}^2$ )

$\text{Re}$  = Reynolds number =  $\frac{v_s \rho d}{\mu}$

$d_{ij}$ ,  $d$  = diameter of filter media particles; arithmetic average of adjacent screen openings (m)

$i$  = filter media (sand, anthracite, garnet)

$j$  = filter media particle size

$x_{ij}$  = mass fraction of media retained between adjacent sieves

$f'_{ij}$  = friction factors for each media fraction

$C_D$  = drag coefficient as defined in settling velocity equations

## Bed Expansion

Monosized

$$L_f = \frac{L_o (1 - \eta_o)}{1 - \left( \frac{v_B}{v_t} \right)^{0.22}}$$

Multisized

$$L_f = L_o (1 - \eta_o) \sum \frac{x_{ij}}{1 - \left( \frac{v_B}{v_{t,ij}} \right)^{0.22}}$$

$$\eta_f = \left( \frac{v_B}{v_t} \right)^{0.22}$$

where

$L_f$  = depth of fluidized filter media (m)

$v_B$  = backwash velocity (m/s),  $Q_B/A_{\text{plan}}$

$Q_B$  = backwash flowrate

$v_t$  = terminal setting velocity

$\eta_f$  = porosity of fluidized bed

$L_o$  = initial bed depth

$\eta_o$  = initial bed porosity

## Lime-Soda Softening Equations

- Carbon dioxide removal  

$$\text{CO}_2 + \text{Ca(OH)}_2 \rightarrow \text{CaCO}_3(\text{s}) + \text{H}_2\text{O}$$
- Calcium carbonate hardness removal  

$$\text{Ca(HCO}_3)_2 + \text{Ca(OH)}_2 \rightarrow 2\text{CaCO}_3(\text{s}) + 2\text{H}_2\text{O}$$
- Calcium non-carbonate hardness removal  

$$\text{CaSO}_4 + \text{Na}_2\text{CO}_3 \rightarrow \text{CaCO}_3(\text{s}) + 2\text{Na}^+ + \text{SO}_4^{2-}$$
- Magnesium carbonate hardness removal  

$$\text{Mg(HCO}_3)_2 + 2\text{Ca(OH)}_2 \rightarrow 2\text{CaCO}_3(\text{s}) + \text{Mg(OH)}_2(\text{s}) + 2\text{H}_2\text{O}$$
- Magnesium non-carbonate hardness removal  

$$\text{MgSO}_4 + \text{Ca(OH)}_2 + \text{Na}_2\text{CO}_3 \rightarrow \text{CaCO}_3(\text{s}) + \text{Mg(OH)}_2(\text{s}) + 2\text{Na}^+ + \text{SO}_4^{2-}$$
- Destruction of excess alkalinity  

$$2\text{HCO}_3^- + \text{Ca(OH)}_2 \rightarrow \text{CaCO}_3(\text{s}) + \text{CO}_3^{2-} + 2\text{H}_2\text{O}$$
- Recarbonation  

$$\text{Ca}^{2+} + 2\text{OH}^- + \text{CO}_2 \rightarrow \text{CaCO}_3(\text{s}) + \text{H}_2\text{O}$$

Molecular Formulas	Molecular Weight	<i>n</i>	Equivalent Weight
		# Equiv per mole	
CO <sub>3</sub> <sup>2-</sup>	60.0	2	30.0
CO <sub>2</sub>	44.0	2	22.0
Ca(OH) <sub>2</sub>	74.1	2	37.1
CaCO <sub>3</sub>	100.1	2	50.0
Ca(HCO <sub>3</sub> ) <sub>2</sub>	162.1	2	81.1
CaSO <sub>4</sub>	136.1	2	68.1
Ca <sup>2+</sup>	40.1	2	20.0
H <sup>+</sup>	1.0	1	1.0
HCO <sub>3</sub> <sup>-</sup>	61.0	1	61.0
Mg(HCO <sub>3</sub> ) <sub>2</sub>	146.3	2	73.2
Mg(OH) <sub>2</sub>	58.3	2	29.2
MgSO <sub>4</sub>	120.4	2	60.2
Mg <sup>2+</sup>	24.3	2	12.2
Na <sup>+</sup>	23.0	1	23.0
Na <sub>2</sub> CO <sub>3</sub>	106.0	2	53.0
OH <sup>-</sup>	17.0	1	17.0
SO <sub>4</sub> <sup>2-</sup>	96.1	2	48.0

## Rapid Mix and Flocculator Design

$$G = \sqrt{\frac{P}{\mu V}} = \sqrt{\frac{\gamma H_L}{t \mu}}$$

$$Gt = 10^4 \text{ to } 10^5$$

where

$G$  = root mean square velocity gradient (mixing intensity)  
 [ft/(sec-ft) or m/(s•m)]

$P$  = power to the fluid (ft-lb/sec or N•m/s)

$V$  = volume (ft<sup>3</sup> or m<sup>3</sup>)

$\mu$  = dynamic viscosity [lb/(ft-sec) or Pa•s]

$\gamma$  = specific weight of water (lb/ft<sup>3</sup> or N/m<sup>3</sup>)

$H_L$  = head loss (ft or m)

$t$  = time (sec or s)

## Reel and Paddle

$$P = \frac{C_D A_p \rho_f v_r^3}{2}$$

where

$C_D$  = drag coefficient = 1.8 for flat blade with a L:W > 20:1

$A_p$  = area of blade (m<sup>2</sup>) perpendicular to the direction of travel through the water

$\rho_f$  = density of H<sub>2</sub>O (kg/m<sup>3</sup>)

$v_p$  = velocity of paddle (m/s)

$v_r$  = relative or effective paddle velocity

=  $v_p$  • slip coefficient

slip coefficient = 0.5 to 0.75

## Turbulent Flow Impeller Mixer

$$P = K_T (n)^3 (D_i)^5 \rho_f$$

where

$K_T$  = impeller constant (see table)

$n$  = rotational speed (rev/sec)

$D_i$  = impeller diameter (m)

## Values of the Impeller Constant $K_T$ (Assume Turbulent Flow)

Type of Impeller	$K_T$
Propeller, pitch of 1, 3 blades	0.32
Propeller, pitch of 2, 3 blades	1.00
Turbine, 6 flat blades, vaned disc	6.30
Turbine, 6 curved blades	4.80
Fan turbine, 6 blades at 45°	1.65
Shrouded turbine, 6 curved blades	1.08
Shrouded turbine, with stator, no baffles	1.12

*Note:* Constant assumes baffled tanks having four baffles at the tank wall with a width equal to 10% of the tank diameter.

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## Reverse Osmosis

### Osmotic Pressure of Solutions of Electrolytes

$$\Pi = \phi v \frac{n}{V} RT$$

where

$\Pi$  = osmotic pressure, Pa

$\phi$  = osmotic coefficient

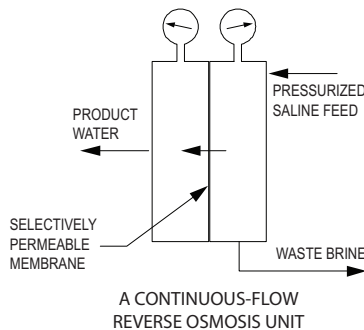
$v$  = number of ions formed from one molecule of electrolyte

$n$  = number of moles of electrolyte

$V$  = specific volume of solvent, m<sup>3</sup>/kmol

$R$  = universal gas constant, Pa • m<sup>3</sup>/(kmol • K)

$T$  = absolute temperature, K



### Salt Flux through the Membrane

$$J_s = (D_s K_s / \Delta Z)(C_{in} - C_{out})$$

where

$J_s$  = salt flux through the membrane [kmol/(m<sup>2</sup> • s)]

$D_s$  = diffusivity of the solute in the membrane (m<sup>2</sup>/s)

$K_s$  = solute distribution coefficient (dimensionless)

$C$  = concentration (kmol/m<sup>3</sup>)

$\Delta Z$  = membrane thickness (m)

$$J_s = K_p (C_{in} - C_{out})$$

$K_p$  = membrane solute mass transfer coefficient

$$= \frac{D_s K_s}{\Delta Z} (L/t, m/s)$$

### Water Flux

$$J_w = W_p (\Delta P - \Delta \pi)$$

where

$J_w$  = water flux through the membrane [kmol/(m<sup>2</sup> • s)]

$W_p$  = coefficient of water permeation, a characteristic of the particular membrane [kmol/(m<sup>2</sup> • s • Pa)]

$\Delta P$  = pressure differential across membrane =  $P_{in} - P_{out}$  (Pa)

$\Delta \pi$  = osmotic pressure differential across membrane

$$\pi_{in} - \pi_{out} (Pa)$$

## Ultrafiltration

$$J_w = \frac{\epsilon r^2 \int \Delta P}{8 \mu \delta}$$

where

$\epsilon$  = membrane porosity

$r$  = membrane pore size

$\Delta P$  = net transmembrane pressure

$\mu$  = viscosity

$\delta$  = membrane thickness

$J_w$  = volumetric flux (m/s)

### Disinfection

Chlorine contact chamber length:width ratio = 20:1 to 50:1

$$T = TDT \times BF$$

$T$  = time that the water is in contact with the disinfectant (min)

$TDT$  = theoretical detention time (min)

$$TDT (\text{min-mg/L}) = C \times T$$

$BF$  = baffling factor

$C$  = residual disinfectant concentration measured during peak hourly flow (mg/L)

◆

### Baffling Factors

Baffling Condition	Baffling Factor	Baffling Description
Unbaffled (mixed flow)	0.1	None, agitated basin, very low length to width ratio, high inlet and outlet flow velocities.
Poor	0.3	Single or multiple unbaffled inlets and outlets, no intra-basin baffles.
Average	0.5	Baffled inlet or outlet with some intra-basin baffles.
Superior	0.7	Perforated inlet baffle, serpentine or perforated intra-basin baffles, outlet weir or perforated launders.
Perfect (plug flow)	1.0	Very high length to width ratio (pipeline flow), perforated inlet, outlet, and intra-basin baffles.

◆ *Guidance Manual LT1ESWTR Disinfection Profiling and Benchmarking*, U.S. Environmental Protection Agency, 2003.

◆ **Removal and Inactivation Requirements**

Microorganism	Required Log Reduction	Treatment
<i>Giardia</i>	3-log (99.9%)	Removal and/or inactivation
Viruses	4-log (99.99%)	Removal and/or inactivation
<i>Cryptosporidium</i>	2-log (99%)	Removal

◆ **Typical Removal Credits and Inactivation Requirements for Various Treatment Technologies**

Process	Typical Log Removal Credits		Resulting Disinfection Log Inactivation Requirements	
	<i>Giardia</i>	Viruses	<i>Giardia</i>	Viruses
Conventional Treatment	2.5	2.0	0.5	2.0
Direct Filtration	2.0	1.0	1.0	3.0
Slow Sand Filtration	2.0	2.0	1.0	2.0
Diatomaceous Earth Filtration	2.0	1.0	1.0	3.0
Unfiltered	0	0	3.0	4.0

◆ *Guidance Manual LT1ESWTR Disinfection Profiling and Benchmarking*, U.S. Environmental Protection Agency, 2003.

### CT Values\* For 3-LOG Inactivation Of *Giardia* Cysts By Free Chlorine

Chlorine Concentration (mg/L)	Temperature <= 0.5°C							Temperature = 5°C							Temperature = 10°C						
	pH							pH							pH						
	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0
<=0.4	137	163	195	237	277	329	390	97	117	139	166	198	236	279	73	88	104	125	149	177	209
0.6	141	168	200	239	286	342	407	100	120	143	171	204	244	291	75	90	107	128	153	183	218
0.8	145	172	205	246	295	354	422	103	122	146	175	210	252	301	78	92	110	131	158	189	226
1.0	148	176	210	253	304	365	437	105	125	149	179	216	260	312	79	94	112	134	162	195	234
1.2	152	180	215	259	313	376	451	107	127	152	183	221	267	320	80	95	114	137	166	200	240
1.4	155	184	221	266	321	387	464	109	130	155	187	227	274	329	82	98	116	140	170	206	247
1.6	157	189	226	273	329	397	477	111	132	158	192	232	281	337	83	99	119	144	174	211	253
1.8	162	193	231	279	338	407	489	114	135	162	196	238	287	345	86	101	122	147	179	215	259
2.0	165	197	236	286	346	417	500	116	138	165	200	243	294	353	87	104	124	150	182	221	265
2.2	169	201	242	297	353	426	511	118	140	169	204	248	300	361	89	105	127	153	186	225	271
2.4	172	205	247	298	361	435	522	120	143	172	209	253	306	368	90	107	129	157	190	230	276
2.6	175	209	252	304	368	444	533	122	146	175	213	258	312	375	92	110	131	160	194	234	281
2.8	178	213	257	310	375	452	543	124	148	178	217	263	318	382	93	111	134	163	197	239	287
3.0	181	217	261	316	382	460	552	126	151	182	221	268	324	389	95	113	137	166	201	243	292

Chlorine Concentration (mg/L)	Temperature = 15°C							Temperature = 20°C							Temperature = 25°C						
	pH							pH							pH						
	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0	<=6.0	6.5	7.0	7.5	8.0	8.5	9.0
<=0.4	49	59	70	83	99	118	140	36	44	52	62	74	89	105	24	29	35	42	50	59	70
0.6	50	60	72	86	102	122	146	38	45	54	64	77	92	109	25	30	36	43	51	61	73
0.8	52	61	73	88	105	126	151	39	46	55	66	79	95	113	26	31	37	44	53	63	75
1.0	53	63	75	90	108	130	156	39	47	56	67	81	98	117	26	31	37	45	54	65	78
1.2	54	64	76	92	111	134	160	40	48	57	69	83	100	120	27	32	38	46	55	67	80
1.4	55	65	78	94	114	137	165	41	49	58	70	85	103	123	27	33	39	47	57	69	82
1.6	56	66	79	96	116	141	169	42	50	59	72	87	105	126	28	33	40	48	58	70	84
1.8	57	68	81	98	119	144	173	43	51	61	74	89	106	129	29	34	41	49	60	72	86
2.0	58	69	83	100	122	147	177	44	52	62	75	91	110	132	29	35	41	50	61	74	88
2.2	59	70	85	102	124	150	181	44	53	63	77	93	113	135	30	35	42	51	62	75	90
2.4	60	72	86	105	127	153	184	45	54	65	78	95	115	138	30	36	43	52	63	77	92
2.6	61	73	88	107	129	156	188	46	55	66	80	97	117	141	31	37	44	53	65	78	94
2.8	62	74	89	109	132	159	191	47	56	67	81	99	119	143	31	37	45	54	66	80	96
3.0	63	76	91	111	134	162	195	47	57	68	83	101	122	146	32	38	46	55	67	81	97

\*Although units did not appear in the original tables, units are min-mg/L

### CT VALUES\* FOR 4-LOG INACTIVATION OF VIRUSES BY FREE CHLORINE

Temperature (°C)	pH	
	6-9	10
0.5	12	90
5	8	60
10	6	45
15	4	30
20	3	22
25	2	15

\*Although units did not appear in the original tables, units are min-mg/L

◆ Guidance Manual LT1ESWTR Disinfection Profiling and Benchmarking, U.S. Environmental Protection Agency, 2003.

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## **APPENDIX**

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**Fundamentals of Engineering (FE)  
CHEMICAL CBT Exam Specifications**

**Effective Beginning with the January 2014 Examinations**

- The FE exam is a computer-based test (CBT). It is closed book with an electronic reference.
- Examinees have 6 hours to complete the exam, which contains 110 multiple-choice questions. The 6-hour time also includes a tutorial, a break, and a brief survey at the conclusion.
- The FE exam uses both the International System of Units (SI) and the US Customary System (USCS).

Knowledge	Number of Questions
<b>1. Mathematics</b> A. Analytic geometry B. Roots of equations C. Calculus D. Differential equations	<b>8–12</b>
<b>2. Probability and Statistics</b> A. Probability distributions (e.g., discrete, continuous, normal, binomial) B. Expected value (weighted average) in decision making C. Hypothesis testing D. Measures of central tendencies and dispersions (e.g., mean, mode, standard deviation) E. Estimation for a single mean (e.g., point, confidence intervals) F. Regression and curve fitting	<b>4–6</b>
<b>3. Engineering Sciences</b> A. Applications of vector analysis (e.g., statics) B. Basic dynamics (e.g., friction, force, mass, acceleration, momentum) C. Work, energy, and power (as applied to particles or rigid bodies) D. Electricity and current and voltage laws (e.g., charge, energy, current, voltage, power, Kirchhoff, Ohm)	<b>4–6</b>
<b>4. Computational Tools</b> A. Numerical methods and concepts (e.g., convergence, tolerance) B. Spreadsheets for chemical engineering calculations C. Simulators	<b>4–6</b>
<b>5. Materials Science</b> A. Chemical, electrical, mechanical, and physical properties (e.g., effect of temperature, pressure, stress, strain) B. Material types and compatibilities (e.g., engineered materials, ferrous and nonferrous metals) C. Corrosion mechanisms and control	<b>4–6</b>

<b>6. Chemistry</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Inorganic chemistry (e.g., molarity, normality, molality, acids, bases, redox reactions, valence, solubility product, pH, pK, electrochemistry, periodic table)</li> <li>B. Organic chemistry (e.g., nomenclature, structure, qualitative and quantitative analyses, balanced equations, reactions, synthesis, basic biochemistry)</li> </ul>	
<b>7. Fluid Mechanics/Dynamics</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Fluid properties</li> <li>B. Dimensionless numbers (e.g., Reynolds number)</li> <li>C. Mechanical energy balance (e.g., pipes, valves, fittings, pressure losses across packed beds, pipe networks)</li> <li>D. Bernoulli equation (hydrostatic pressure, velocity head)</li> <li>E. Laminar and turbulent flow</li> <li>F. Flow measurement (e.g., orifices, Venturi meters)</li> <li>G. Pumps, turbines, and compressors</li> <li>H. Compressible flow and non-Newtonian fluids</li> </ul>	
<b>8. Thermodynamics</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Thermodynamic properties (e.g. specific volume, internal energy, enthalpy, entropy, free energy)</li> <li>B. Properties data and phase diagrams (e.g. steam tables, psychrometric charts, T-s, P-h, x-y, T-x-y)</li> <li>C. Thermodynamic laws (e.g., 1st law, 2nd law)</li> <li>D. Thermodynamic processes (e.g., isothermal, adiabatic, isentropic)</li> <li>E. Cyclic processes and efficiency (e.g., power, refrigeration, heat pump)</li> <li>F. Phase equilibrium (e.g., fugacity, activity coefficient)</li> <li>G. Chemical equilibrium</li> <li>H. Heats of reaction and mixing</li> </ul>	
<b>9. Material/Energy Balances</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Mass balance (steady and unsteady state)</li> <li>B. Energy balance (steady and unsteady state)</li> <li>C. Recycle/bypass processes</li> <li>D. Reactive systems (e.g., combustion)</li> </ul>	
<b>10. Heat Transfer</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Conductive heat transfer</li> <li>B. Convective heat transfer (natural and forced)</li> <li>C. Radiation heat transfer</li> <li>D. Heat transfer coefficients (e.g., overall, local, fouling)</li> <li>E. Heat transfer equipment, operation, and design (e.g., double pipe, shell and tube, fouling, number of transfer units, log-mean temperature difference, flow configuration)</li> </ul>	
<b>11. Mass Transfer and Separation</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Molecular diffusion (e.g., steady and unsteady state, physical property estimation)</li> <li>B. Convective mass transfer (e.g., mass transfer coefficient, eddy diffusion)</li> <li>C. Separation systems (e.g., distillation, absorption, extraction, membrane processes)</li> </ul>	



- D. Equilibrium stage methods (e.g., graphical methods, McCabe-Thiele, efficiency)
- E. Continuous contact methods (e.g., number of transfer units, height equivalent to a theoretical plate, height of transfer unit, number of theoretical plates)
- F. Humidification and drying

**12. Chemical Reaction Engineering** **8–12**

- A. Reaction rates and order
- B. Rate constant (e.g., Arrhenius function)
- C. Conversion, yield, and selectivity
- D. Type of reactions (e.g., series, parallel, forward, reverse, homogeneous, heterogeneous, catalysis, biocatalysis)
- E. Reactor types (e.g., batch, semibatch, continuous stirred tank, plug flow, gas phase, liquid phase)

**13. Process Design and Economics** **8–12**

- A. Process flow diagrams and piping and instrumentation diagrams
- B. Equipment selection (e.g., sizing and scale-up)
- C. Cost estimation
- D. Comparison of economic alternatives (e.g., net present value, discounted cash flow, rate of return, expected value and risk)
- E. Process design and optimization (e.g., sustainability, efficiency, green engineering, inherently safer design, evaluation of specifications)

**14. Process Control** **5–8**

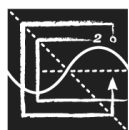
- A. Dynamics (e.g., time constants and 2nd order, underdamped, and transfer functions)
- B. Control strategies (e.g., feedback, feed-forward, cascade, ratio, and PID)
- C. Control loop design and hardware (e.g., matching measured and manipulated variables, sensors, control valves, and conceptual process control)

**15. Safety, Health, and Environment** **5–8**

- A. Hazardous properties of materials (e.g., corrosivity, flammability, toxicity, reactivity, handling and storage), including MSDS
- B. Industrial hygiene (e.g., noise, PPE, ergonomics)
- C. Process safety and hazard analysis [e.g., layer of protection analysis, hazard and operability studies (HazOps), fault-tree analysis or event tree]
- D. Overpressure and underpressure protection (e.g., relief, redundant control, intrinsically safe)
- E. Waste minimization, waste treatment, and regulation (e.g., air, water, solids, RCRA, CWA, EPA, OSHA)

**16. Ethics and Professional Practice** **2–3**

- A. Codes of ethics (professional and technical societies)
- B. Agreements and contracts
- C. Ethical and legal considerations
- D. Professional liability
- E. Public protection issues (e.g., licensing boards)



## Fundamentals of Engineering (FE) CIVIL CBT Exam Specifications

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- The FE exam is a computer-based test (CBT). It is closed book with an electronic reference.
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- The FE exam uses both the International System of Units (SI) and the US Customary System (USCS).

Knowledge	Number of Questions
<b>1. Mathematics</b>	<b>7–11</b>
A. Analytic geometry	
B. Calculus	
C. Roots of equations	
D. Vector analysis	
<b>2. Probability and Statistics</b>	<b>4–6</b>
A. Measures of central tendencies and dispersions (e.g., mean, mode, standard deviation)	
B. Estimation for a single mean (e.g., point, confidence intervals)	
C. Regression and curve fitting	
D. Expected value (weighted average) in decision making	
<b>3. Computational Tools</b>	<b>4–6</b>
A. Spreadsheet computations	
B. Structured programming (e.g., if-then, loops, macros)	
<b>4. Ethics and Professional Practice</b>	<b>4–6</b>
A. Codes of ethics (professional and technical societies)	
B. Professional liability	
C. Licensure	
D. Sustainability and sustainable design	
E. Professional skills (e.g., public policy, management, and business)	
F. Contracts and contract law	
<b>5. Engineering Economics</b>	<b>4–6</b>
A. Discounted cash flow (e.g., equivalence, PW, equivalent annual worth, FW, rate of return)	
B. Cost (e.g., incremental, average, sunk, estimating)	
C. Analyses (e.g., breakeven, benefit-cost, life cycle)	
D. Uncertainty (e.g., expected value and risk)	
<b>6. Statics</b>	<b>7–11</b>
A. Resultants of force systems	
B. Equivalent force systems	
C. Equilibrium of rigid bodies	
D. Frames and trusses	

E. Centroid of area	
F. Area moments of inertia	
G. Static friction	
<b>7. Dynamics</b>	<b>4–6</b>
A. Kinematics (e.g., particles and rigid bodies)	
B. Mass moments of inertia	
C. Force acceleration (e.g., particles and rigid bodies)	
D. Impulse momentum (e.g., particles and rigid bodies)	
E. Work, energy, and power (e.g., particles and rigid bodies)	
<b>8. Mechanics of Materials</b>	<b>7–11</b>
A. Shear and moment diagrams	
B. Stresses and strains (e.g., axial, torsion, bending, shear, thermal)	
C. Deformations (e.g., axial, torsion, bending, thermal)	
D. Combined stresses	
E. Principal stresses	
F. Mohr's circle	
G. Column analysis (e.g., buckling, boundary conditions)	
H. Composite sections	
I. Elastic and plastic deformations	
J. Stress-strain diagrams	
<b>9. Materials</b>	<b>4–6</b>
A. Mix design (e.g., concrete and asphalt)	
B. Test methods and specifications (e.g., steel, concrete, aggregates, asphalt, wood)	
C. Physical and mechanical properties of concrete, ferrous and nonferrous metals, masonry, wood, engineered materials (e.g., FRP, laminated lumber, wood/plastic composites), and asphalt	
<b>10. Fluid Mechanics</b>	<b>4–6</b>
A. Flow measurement	
B. Fluid properties	
C. Fluid statics	
D. Energy, impulse, and momentum equations	
<b>11. Hydraulics and Hydrologic Systems</b>	<b>8–12</b>
A. Basic hydrology (e.g., infiltration, rainfall, runoff, detention, flood flows, watersheds)	
B. Basic hydraulics (e.g., Manning equation, Bernoulli theorem, open-channel flow, pipe flow)	
C. Pumping systems (water and wastewater)	
D. Water distribution systems	
E. Reservoirs (e.g., dams, routing, spillways)	
F. Groundwater (e.g., flow, wells, drawdown)	
G. Storm sewer collection systems	
<b>12. Structural Analysis</b>	<b>6–9</b>
A. Analysis of forces in statically determinant beams, trusses, and frames	
B. Deflection of statically determinant beams, trusses, and frames	
C. Structural determinacy and stability analysis of beams, trusses, and frames	

- D. Loads and load paths (e.g., dead, live, lateral, influence lines and moving loads, tributary areas)
- E. Elementary statically indeterminate structures

**13. Structural Design** **6–9**

- A. Design of steel components (e.g., codes and design philosophies, beams, columns, beam-columns, tension members, connections)
- B. Design of reinforced concrete components (e.g., codes and design philosophies, beams, slabs, columns, walls, footings)

**14. Geotechnical Engineering** **9–14**

- A. Geology
- B. Index properties and soil classifications
- C. Phase relations (air-water-solid)
- D. Laboratory and field tests
- E. Effective stress (buoyancy)
- F. Stability of retaining walls (e.g., active pressure/passive pressure)
- G. Shear strength
- H. Bearing capacity (cohesive and noncohesive)
- I. Foundation types (e.g., spread footings, deep foundations, wall footings, mats)
- J. Consolidation and differential settlement
- K. Seepage/flow nets
- L. Slope stability (e.g., fills, embankments, cuts, dams)
- M. Soil stabilization (e.g., chemical additives, geosynthetics)
- N. Drainage systems
- O. Erosion control

**15. Transportation Engineering** **8–12**

- A. Geometric design of streets and highways
- B. Geometric design of intersections
- C. Pavement system design (e.g., thickness, subgrade, drainage, rehabilitation)
- D. Traffic safety
- E. Traffic capacity
- F. Traffic flow theory
- G. Traffic control devices
- H. Transportation planning (e.g., travel forecast modeling)

**16. Environmental Engineering** **6–9**

- A. Water quality (ground and surface)
- B. Basic tests (e.g., water, wastewater, air)
- C. Environmental regulations
- D. Water supply and treatment
- E. Wastewater collection and treatment

**17. Construction**

**4–6**

- A. Construction documents
- B. Procurement methods (e.g., competitive bid, qualifications-based)
- C. Project delivery methods (e.g., design-bid-build, design build, construction management, multiple prime)
- D. Construction operations and methods (e.g., lifting, rigging, dewatering and pumping, equipment production, productivity analysis and improvement, temporary erosion control)
- E. Project scheduling (e.g., CPM, allocation of resources)
- F. Project management (e.g., owner/contractor/client relations)
- G. Construction safety
- H. Construction estimating

**18. Surveying**

**4–6**

- A. Angles, distances, and trigonometry
- B. Area computations
- C. Earthwork and volume computations
- D. Closure
- E. Coordinate systems (e.g., state plane, latitude/longitude)
- F. Leveling (e.g., differential, elevations, percent grades)

**Fundamentals of Engineering (FE)  
ELECTRICAL AND COMPUTER CBT Exam Specifications  
Effective Beginning with the January 2014 Examinations**

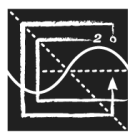
- The FE exam is a computer-based test (CBT). It is closed book with an electronic reference.
- Examinees have 6 hours to complete the exam, which contains 110 multiple-choice questions. The 6-hour time also includes a tutorial, a break, and a brief survey at the conclusion.
- The FE exam uses both the International System of Units (SI) and the US Customary System (USCS).

Knowledge	Number of Questions
<b>1. Mathematics</b> A. Algebra and trigonometry B. Complex numbers C. Discrete mathematics D. Analytic geometry E. Calculus F. Differential equations G. Linear algebra H. Vector analysis	<b>11–17</b>
<b>2. Probability and Statistics</b> A. Measures of central tendencies and dispersions (e.g., mean, mode, standard deviation) B. Probability distributions (e.g., discrete, continuous, normal, binomial) C. Expected value (weighted average) in decision making D. Estimation for a single mean (e.g., point, confidence intervals, conditional probability)	<b>4–6</b>
<b>3. Ethics and Professional Practice</b> A. Codes of ethics (professional and technical societies) B. NCEES Model Law and Model Rules C. Intellectual property (e.g., copyright, trade secrets, patents)	<b>3–5</b>
<b>4. Engineering Economics</b> A. Time value of money (e.g., present value, future value, annuities) B. Cost estimation C. Risk identification D. Analysis (e.g., cost-benefit, trade-off, breakeven)	<b>3–5</b>
<b>5. Properties of Electrical Materials</b> A. Chemical (e.g., corrosion, ions, diffusion) B. Electrical (e.g., conductivity, resistivity, permittivity, magnetic permeability) C. Mechanical (e.g., piezoelectric, strength) D. Thermal (e.g., conductivity, expansion)	<b>4–6</b>

<b>6. Engineering Sciences</b>	<b>6–9</b>
<ul style="list-style-type: none"> <li>A. Work, energy, power, heat</li> <li>B. Charge, energy, current, voltage, power</li> <li>C. Forces (e.g., between charges, on conductors)</li> <li>D. Work done in moving a charge in an electric field (relationship between voltage and work)</li> <li>E. Capacitance</li> <li>F. Inductance</li> </ul>	
<b>7. Circuit Analysis (DC and AC Steady State)</b>	<b>10–15</b>
<ul style="list-style-type: none"> <li>A. KCL, KVL</li> <li>B. Series/parallel equivalent circuits</li> <li>C. Thevenin and Norton theorems</li> <li>D. Node and loop analysis</li> <li>E. Waveform analysis (e.g., RMS, average, frequency, phase, wavelength)</li> <li>F. Phasors</li> <li>G. Impedance</li> </ul>	
<b>8. Linear Systems</b>	<b>5–8</b>
<ul style="list-style-type: none"> <li>A. Frequency/transient response</li> <li>B. Resonance</li> <li>C. Laplace transforms</li> <li>D. Transfer functions</li> <li>E. 2-port theory</li> </ul>	
<b>9. Signal Processing</b>	<b>5–8</b>
<ul style="list-style-type: none"> <li>A. Convolution (continuous and discrete)</li> <li>B. Difference equations</li> <li>C. Z-transforms</li> <li>D. Sampling (e.g., aliasing, Nyquist theorem)</li> <li>E. Analog filters</li> <li>F. Digital filters</li> </ul>	
<b>10. Electronics</b>	<b>7–11</b>
<ul style="list-style-type: none"> <li>A. Solid-state fundamentals (e.g., tunneling, diffusion/drift current, energy bands, doping bands, p-n theory)</li> <li>B. Discrete devices (diodes, transistors, BJT, CMOS) and models and their performance</li> <li>C. Bias circuits</li> <li>D. Amplifiers (e.g., single-stage/common emitter, differential)</li> <li>E. Operational amplifiers (ideal, non-ideal)</li> <li>F. Instrumentation (e.g., measurements, data acquisition, transducers)</li> <li>G. Power electronics</li> </ul>	
<b>11. Power</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Single phase and three phase</li> <li>B. Transmission and distribution</li> <li>C. Voltage regulation</li> <li>D. Transformers</li> <li>E. Motors and generators</li> <li>F. Power factor (pf)</li> </ul>	

<b>12. Electromagnetics</b>	<b>5–8</b>
A. Maxwell equations	
B. Electrostatics/magnetostatics (e.g., measurement of spatial relationships, vector analysis)	
C. Wave propagation	
D. Transmission lines (high frequency)	
E. Electromagnetic compatibility	
<b>13. Control Systems</b>	<b>6–9</b>
A. Block diagrams (feed-forward, feedback)	
B. Bode plots	
C. Closed-loop and open-loop response	
D. Controller performance (gain, PID), steady-state errors	
E. Root locus	
F. Stability	
G. State variables	
<b>14. Communications</b>	<b>5–8</b>
A. Basic modulation/demodulation concepts (e.g., AM, FM, PCM)	
B. Fourier transforms/Fourier series	
C. Multiplexing (e.g., time division, frequency division)	
D. Digital communications	
<b>15. Computer Networks</b>	<b>3–5</b>
A. Routing and switching	
B. Network topologies/frameworks/models	
C. Local area networks	
<b>16. Digital Systems</b>	<b>7–11</b>
A. Number systems	
B. Boolean logic	
C. Logic gates and circuits	
D. Logic minimization (e.g., SOP, POS, Karnaugh maps)	
E. Flip-flops and counters	
F. Programmable logic devices and gate arrays	
G. State machine design	
H. Data path/controller design	
I. Timing (diagrams, asynchronous inputs, races, hazards)	
<b>17. Computer Systems</b>	<b>4–6</b>
A. Architecture (e.g., pipelining, cache memory)	
B. Microprocessors	
C. Memory technology and systems	
D. Interfacing	
<b>18. Software Development</b>	<b>4–6</b>
A. Algorithms	
B. Data structures	
C. Software design methods (structured, object-oriented)	
D. Software implementation (e.g., procedural, scripting languages)	
E. Software testing	





## Fundamentals of Engineering (FE) ENVIRONMENTAL CBT Exam Specifications

Effective Beginning with the January 2014 Examinations

- The FE exam is a computer-based test (CBT). It is closed book with an electronic reference.
- Examinees have 6 hours to complete the exam, which contains 110 multiple-choice questions. The 6-hour time also includes a tutorial, a break, and a brief survey at the conclusion.
- The FE exam uses both the International System of Units (SI) and the US Customary System (USCS).

Knowledge	Number of Questions
<b>1. Mathematics</b>	<b>4–6</b>
A. Analytic geometry	
B. Numerical methods	
C. Roots of equations	
D. Calculus	
E. Differential equations	
<b>2. Probability and Statistics</b>	<b>3–5</b>
A. Measures of central tendencies and dispersions (e.g., mean, mode, standard deviation)	
B. Probability distributions (e.g., discrete, continuous, normal, binomial)	
C. Estimation (point, confidence intervals) for a single mean	
D. Regression and curve fitting	
E. Expected value (weighted average) in decision making	
F. Hypothesis testing	
<b>3. Ethics and Professional Practice</b>	<b>5–8</b>
A. Codes of ethics (professional and technical societies)	
B. Agreements and contracts	
C. Ethical and legal considerations	
D. Professional liability	
E. Public protection issues (e.g., licensing boards)	
F. Regulations (e.g., water, wastewater, air, solid/hazardous waste, groundwater/soils)	
<b>4. Engineering Economics</b>	<b>4–6</b>
A. Discounted cash flow (e.g., life cycle, equivalence, PW, equivalent annual worth, FW, rate of return)	
B. Cost (e.g., incremental, average, sunk, estimating)	
C. Analyses (e.g., breakeven, benefit-cost)	
D. Uncertainty (expected value and risk)	
<b>5. Materials Science</b>	<b>3–5</b>
A. Properties (e.g., chemical, electrical, mechanical, physical)	
B. Corrosion mechanisms and controls	
C. Material selection and compatibility	

<b>6. Environmental Science and Chemistry</b>	<b>11–17</b>
A. Reactions (e.g., equilibrium, acid base, oxidation-reduction, precipitation)	
B. Stoichiometry	
C. Kinetics (chemical, microbiological)	
D. Organic chemistry (e.g., nomenclature, functional group reactions)	
E. Ecology (e.g., Streeter-Phelps, fluviology, limnology, eutrophication)	
F. Multimedia equilibrium partitioning (e.g., Henry’s law, octonal partitioning coefficient)	
<b>7. Risk Assessment</b>	<b>5–8</b>
A. Dose-response toxicity (carcinogen, noncarcinogen)	
B. Exposure routes	
<b>8. Fluid Mechanics</b>	<b>9–14</b>
A. Fluid statics	
B. Closed conduits (e.g., Darcy-Weisbach, Hazen-Williams, Moody)	
C. Open channel (Manning)	
D. Pumps (e.g., power, operating point, parallel and series)	
E. Flow measurement (e.g., weirs, orifices, flowmeters)	
F. Blowers (e.g., power, operating point, parallel, and series)	
<b>9. Thermodynamics</b>	<b>3–5</b>
A. Thermodynamic laws (e.g., 1st law, 2nd law)	
B. Energy, heat, and work	
C. Ideal gases	
D. Mixture of nonreacting gases	
E. Heat transfer	
<b>10. Water Resources</b>	<b>10–15</b>
A. Demand calculations	
B. Population estimations	
C. Runoff calculations (e.g., land use, land cover, time of concentration, duration, intensity, frequency)	
D. Reservoir sizing	
E. Routing (e.g., channel, reservoir)	
F. Water quality and modeling (e.g., erosion, channel stability, stormwater quality management)	
<b>11. Water and Wastewater</b>	<b>14–21</b>
A. Water and wastewater characteristics	
B. Mass and energy balances	
C. Conventional water treatment processes (e.g., clarification, disinfection, filtration, flocculation, softening, rapid mix)	
D. Conventional wastewater treatment processes (e.g., activated sludge, decentralized wastewater systems, fixed-film system, disinfection, flow equalization, headworks, lagoons)	
E. Alternative treatment process (e.g., conservation and reuse, membranes, nutrient removal, ion exchange, activated carbon, air stripping)	
F. Sludge treatment and handling (e.g., land application, sludge digestion, sludge dewatering)	

- 12. Air Quality** **10–15**
- A. Chemical principles (e.g., ideal gas, mole fractions, stoichiometry, Henry's law)
  - B. Mass balances
  - C. Emissions (factors, rates)
  - D. Atmospheric sciences (e.g., stability classes, dispersion modeling, lapse rates)
  - E. Gas handling and treatment technologies (e.g., hoods, ducts, coolers, biofiltration, scrubbers, adsorbers, incineration)
  - F. Particle handling and treatment technologies (e.g., baghouses, cyclones, electrostatic precipitators, settling velocity)
- 13. Solid and Hazardous Waste** **10–15**
- A. Composting
  - B. Mass balances
  - C. Compatibility
  - D. Landfilling (e.g., siting, design, leachate, material and energy recovery)
  - E. Site characterization and remediation
  - F. Hazardous waste treatment (e.g., physical, chemical, thermal)
  - G. Radioactive waste treatment and disposal
- 14. Groundwater and Soils** **9–14**
- A. Basic hydrogeology (e.g., aquifers, permeability, water table, hydraulic conductivity, saturation, soil characteristics)
  - B. Drawdown (e.g., Jacob, Theis, Thiem)
  - C. Groundwater flow (e.g., Darcy's law, specific capacity, velocity, gradient)
  - D. Soil and groundwater remediation

**Fundamentals of Engineering (FE)  
INDUSTRIAL CBT Exam Specifications**

**Effective Beginning with the January 2014 Examinations**

- The FE exam is a computer-based test (CBT). It is closed book with an electronic reference.
- Examinees have 6 hours to complete the exam, which contains 110 multiple-choice questions. The 6-hour time also includes a tutorial, a break, and a brief survey at the conclusion.
- The FE exam uses both the International System of Units (SI) and the US Customary System (USCS).

Knowledge	Number of Questions
<b>1. Mathematics</b> A. Analytic geometry B. Calculus C. Matrix operations D. Vector analysis E. Linear algebra	<b>6–9</b>
<b>2. Engineering Sciences</b> A. Work, energy, and power B. Material properties and selection C. Charge, energy, current, voltage, and power	<b>5–8</b>
<b>3. Ethics and Professional Practice</b> A. Codes of ethics and licensure B. Agreements and contracts C. Professional, ethical, and legal responsibility D. Public protection and regulatory issues	<b>5–8</b>
<b>4. Engineering Economics</b> A. Discounted cash flows (PW, EAC, FW, IRR, amortization) B. Types and breakdown of costs (e.g., fixed, variable, direct and indirect labor) C. Cost analyses (e.g., benefit-cost, breakeven, minimum cost, overhead) D. Accounting (financial statements and overhead cost allocation) E. Cost estimation F. Depreciation and taxes G. Capital budgeting	<b>10–15</b>
<b>5. Probability and Statistics</b> A. Combinatorics (e.g., combinations, permutations) B. Probability distributions (e.g., normal, binomial, empirical) C. Conditional probabilities D. Sampling distributions, sample sizes, and statistics (e.g., central tendency, dispersion) E. Estimation (e.g., point, confidence intervals) F. Hypothesis testing G. Regression (linear, multiple)	<b>10–15</b>

- H. System reliability (e.g., single components, parallel and series systems)
- I. Design of experiments (e.g., ANOVA, factorial designs)

**6. Modeling and Computations 8–12**

- A. Algorithm and logic development (e.g., flow charts, pseudocode)
- B. Databases (e.g., types, information content, relational)
- C. Decision theory (e.g., uncertainty, risk, utility, decision trees)
- D. Optimization modeling (e.g., decision variables, objective functions, and constraints)
- E. Linear programming (e.g., formulation, primal, dual, graphical solutions)
- F. Mathematical programming (e.g., network, integer, dynamic, transportation, assignment)
- G. Stochastic models (e.g., queuing, Markov, reliability)
- H. Simulation

**7. Industrial Management 8–12**

- A. Principles (e.g., planning, organizing, motivational theory)
- B. Tools of management (e.g., MBO, reengineering, organizational structure)
- C. Project management (e.g., scheduling, PERT, CPM)
- D. Productivity measures

**8. Manufacturing, Production, and Service Systems 8–12**

- A. Manufacturing processes
- B. Manufacturing systems (e.g., cellular, group technology, flexible)
- C. Process design (e.g., resources, equipment selection, line balancing)
- D. Inventory analysis (e.g., EOQ, safety stock)
- E. Forecasting
- F. Scheduling (e.g., sequencing, cycle time, material control)
- G. Aggregate planning
- H. Production planning (e.g., JIT, MRP, ERP)
- I. Lean enterprises
- J. Automation concepts (e.g., robotics, CIM)
- K. Sustainable manufacturing (e.g., energy efficiency, waste reduction)
- L. Value engineering

**9. Facilities and Logistics 8–12**

- A. Flow measurements and analysis (e.g., from/to charts, flow planning)
- B. Layouts (e.g., types, distance metrics, planning, evaluation)
- C. Location analysis (e.g., single- and multiple-facility location, warehouses)
- D. Process capacity analysis (e.g., number of machines and people, trade-offs)
- E. Material handling capacity analysis
- F. Supply chain management and design

**10. Human Factors, Ergonomics, and Safety 8–12**

- A. Hazard identification and risk assessment
- B. Environmental stress assessment (e.g., noise, vibrations, heat)
- C. Industrial hygiene
- D. Design for usability (e.g., tasks, tools, displays, controls, user interfaces)
- E. Anthropometry
- F. Biomechanics
- G. Cumulative trauma disorders (e.g., low back injuries, carpal tunnel syndrome)

- H. Systems safety
- I. Cognitive engineering (e.g., information processing, situation awareness, human error, mental models)

**11. Work Design 8–12**

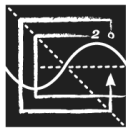
- A. Methods analysis (e.g., charting, workstation design, motion economy)
- B. Time study (e.g., time standards, allowances)
- C. Predetermined time standard systems (e.g., MOST, MTM)
- D. Work sampling
- E. Learning curves

**12. Quality 8–12**

- A. Six sigma
- B. Management and planning tools (e.g., fishbone, Pareto, QFD, TQM)
- C. Control charts
- D. Process capability and specifications
- E. Sampling plans
- F. Design of experiments for quality improvement
- G. Reliability engineering

**13. Systems Engineering 8–12**

- A. Requirements analysis
- B. System design
- C. Human systems integration
- D. Functional analysis and allocation
- E. Configuration management
- F. Risk management
- G. Verification and assurance
- H. System life-cycle engineering



# NCEES

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engineers and surveyors*

## Fundamentals of Engineering (FE) MECHANICAL CBT Exam Specifications

Effective Beginning with the January 2014 Examinations

- The FE exam is a computer-based test (CBT). It is closed book with an electronic reference.
- Examinees have 6 hours to complete the exam, which contains 110 multiple-choice questions. The 6-hour time also includes a tutorial, a break, and a brief survey at the conclusion.
- The FE exam uses both the International System of Units (SI) and the US Customary System (USCS).

Knowledge	Number of Questions
<b>1. Mathematics</b> A. Analytic geometry B. Calculus C. Linear algebra D. Vector analysis E. Differential equations F. Numerical methods	<b>6–9</b>
<b>2. Probability and Statistics</b> A. Probability distributions B. Regression and curve fitting	<b>4–6</b>
<b>3. Computational Tools</b> A. Spreadsheets B. Flow charts	<b>3–5</b>
<b>4. Ethics and Professional Practice</b> A. Codes of ethics B. Agreements and contracts C. Ethical and legal considerations D. Professional liability E. Public health, safety, and welfare	<b>3–5</b>
<b>5. Engineering Economics</b> A. Time value of money B. Cost, including incremental, average, sunk, and estimating C. Economic analyses D. Depreciation	<b>3–5</b>
<b>6. Electricity and Magnetism</b> A. Charge, current, voltage, power, and energy B. Current and voltage laws (Kirchhoff, Ohm) C. Equivalent circuits (series, parallel) D. AC circuits E. Motors and generators	<b>3–5</b>

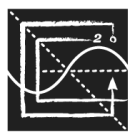
<b>7. Statics</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Resultants of force systems</li> <li>B. Concurrent force systems</li> <li>C. Equilibrium of rigid bodies</li> <li>D. Frames and trusses</li> <li>E. Centroids</li> <li>F. Moments of inertia</li> <li>G. Static friction</li> </ul>	
<b>8. Dynamics, Kinematics, and Vibrations</b>	<b>9–14</b>
<ul style="list-style-type: none"> <li>A. Kinematics of particles</li> <li>B. Kinetic friction</li> <li>C. Newton’s second law for particles</li> <li>D. Work-energy of particles</li> <li>E. Impulse-momentum of particles</li> <li>F. Kinematics of rigid bodies</li> <li>G. Kinematics of mechanisms</li> <li>H. Newton’s second law for rigid bodies</li> <li>I. Work-energy of rigid bodies</li> <li>J. Impulse-momentum of rigid bodies</li> <li>K. Free and forced vibrations</li> </ul>	
<b>9. Mechanics of Materials</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Shear and moment diagrams</li> <li>B. Stress types (axial, bending, torsion, shear)</li> <li>C. Stress transformations</li> <li>D. Mohr’s circle</li> <li>E. Stress and strain caused by axial loads</li> <li>F. Stress and strain caused by bending loads</li> <li>G. Stress and strain caused by torsion</li> <li>H. Stress and strain caused by shear</li> <li>I. Combined loading</li> <li>J. Deformations</li> <li>K. Columns</li> </ul>	
<b>10. Material Properties and Processing</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Properties, including chemical, electrical, mechanical, physical, and thermal</li> <li>B. Stress-strain diagrams</li> <li>C. Engineered materials</li> <li>D. Ferrous metals</li> <li>E. Nonferrous metals</li> <li>F. Manufacturing processes</li> <li>G. Phase diagrams</li> <li>H. Phase transformation, equilibrium, and heat treating</li> <li>I. Materials selection</li> <li>J. Surface conditions</li> <li>K. Corrosion mechanisms and control</li> <li>L. Thermal failure</li> </ul>	



- M. Ductile or brittle behavior
- N. Fatigue
- O. Crack propagation

- |     |  |       |
|-----|--|-------|
| 11. | <b>Fluid Mechanics</b>   | 9–14  |
|     | <ul style="list-style-type: none"> <li>A. Fluid properties</li> <li>B. Fluid statics</li> <li>C. Energy, impulse, and momentum</li> <li>D. Internal flow</li> <li>E. External flow</li> <li>F. Incompressible flow</li> <li>G. Compressible flow</li> <li>H. Power and efficiency</li> <li>I. Performance curves</li> <li>J. Scaling laws for fans, pumps, and compressors</li> </ul>  |       |
| 12. | <b>Thermodynamics</b>  | 13–20 |
|     | <ul style="list-style-type: none"> <li>A. Properties of ideal gases and pure substances</li> <li>B. Energy transfers</li> <li>C. Laws of thermodynamics</li> <li>D. Processes</li> <li>E. Performance of components</li> <li>F. Power cycles, thermal efficiency, and enhancements</li> <li>G. Refrigeration and heat pump cycles and coefficients of performance</li> <li>H. Nonreacting mixtures of gases</li> <li>I. Psychrometrics</li> <li>J. Heating, ventilating, and air-conditioning (HVAC) processes</li> <li>K. Combustion and combustion products</li> </ul> |       |
| 13. | <b>Heat Transfer</b>   | 9–14  |
|     | <ul style="list-style-type: none"> <li>A. Conduction</li> <li>B. Convection</li> <li>C. Radiation</li> <li>D. Thermal resistance</li> <li>E. Transient processes</li> <li>F. Heat exchangers</li> <li>G. Boiling and condensation</li> </ul>   |       |
| 14. | <b>Measurements, Instrumentation, and Controls</b>   | 5–8   |
|     | <ul style="list-style-type: none"> <li>A. Sensors</li> <li>B. Block diagrams</li> <li>C. System response</li> <li>D. Measurement uncertainty</li> </ul>  |       |
| 15. | <b>Mechanical Design and Analysis</b>  | 9–14  |
|     | <ul style="list-style-type: none"> <li>A. Stress analysis of machine elements</li> <li>B. Failure theories and analysis</li> <li>C. Deformation and stiffness</li> <li>D. Springs</li> <li>E. Pressure vessels</li> <li>F. Beams</li> <li>G. Piping</li> </ul>   |       |

- H. Bearings
- I. Power screws
- J. Power transmission
- K. Joining methods
- L. Manufacturability
- M. Quality and reliability
- N. Hydraulic components
- O. Pneumatic components
- P. Electromechanical components



## Fundamentals of Engineering (FE) OTHER DISCIPLINES CBT Exam Specifications

Effective Beginning with the January 2014 Examinations

- The FE exam is a computer-based test (CBT). It is closed book with an electronic reference.
- Examinees have 6 hours to complete the exam, which contains 110 multiple-choice questions. The 6-hour time also includes a tutorial, a break, and a brief survey at the conclusion.
- The FE exam uses both the International System of Units (SI) and the US Customary System (USCS).

Knowledge	Number of Questions
<b>1. Mathematics and Advanced Engineering Mathematics</b>	<b>12–18</b>
A. Analytic geometry and trigonometry	
B. Calculus	
C. Differential equations (e.g., homogeneous, nonhomogeneous, Laplace transforms)	
D. Numerical methods (e.g., algebraic equations, roots of equations, approximations, precision limits)	
E. Linear algebra (e.g., matrix operations)	
<b>2. Probability and Statistics</b>	<b>6–9</b>
A. Measures of central tendencies and dispersions (e.g., mean, mode, variance, standard deviation)	
B. Probability distributions (e.g., discrete, continuous, normal, binomial)	
C. Estimation (e.g., point, confidence intervals)	
D. Expected value (weighted average) in decision making	
E. Sample distributions and sizes	
F. Goodness of fit (e.g., correlation coefficient, least squares)	
<b>3. Chemistry</b>	<b>7–11</b>
A. Periodic table (e.g., nomenclature, metals and nonmetals, atomic structure of matter)	
B. Oxidation and reduction	
C. Acids and bases	
D. Equations (e.g., stoichiometry, equilibrium)	
E. Gas laws (e.g., Boyle's and Charles' Laws, molar volume)	
<b>4. Instrumentation and Data Acquisition</b>	<b>4–6</b>
A. Sensors (e.g., temperature, pressure, motion, pH, chemical constituents)	
B. Data acquisition (e.g., logging, sampling rate, sampling range, filtering, amplification, signal interface)	
C. Data processing (e.g., flow charts, loops, branches)	
<b>5. Ethics and Professional Practice</b>	<b>3–5</b>
A. Codes of ethics	
B. NCEES <i>Model Law</i>	
C. Public protection issues (e.g., licensing boards)	

<b>6. Safety, Health, and Environment</b>	<b>4–6</b>
<ul style="list-style-type: none"> <li>A. Industrial hygiene (e.g., carcinogens, toxicology, MSDS, lower exposure limits)</li> <li>B. Basic safety equipment (e.g., pressure relief valves, emergency shut-offs, fire prevention and control, personal protective equipment)</li> <li>C. Gas detection and monitoring (e.g., O<sub>2</sub>, CO, CO<sub>2</sub>, CH<sub>4</sub>, H<sub>2</sub>S, Radon)</li> <li>D. Electrical safety</li> </ul>	
<b>7. Engineering Economics</b>	<b>7–11</b>
<ul style="list-style-type: none"> <li>A. Time value of money (e.g., present worth, annual worth, future worth, rate of return)</li> <li>B. Cost (e.g., incremental, average, sunk, estimating)</li> <li>C. Economic analyses (e.g., breakeven, benefit-cost, optimal economic life)</li> <li>D. Uncertainty (e.g., expected value and risk)</li> <li>E. Project selection (e.g., comparison of unequal life projects, lease/buy/make, depreciation, discounted cash flow)</li> </ul>	
<b>8. Statics</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Resultants of force systems and vector analysis</li> <li>B. Concurrent force systems</li> <li>C. Force couple systems</li> <li>D. Equilibrium of rigid bodies</li> <li>E. Frames and trusses</li> <li>F. Area properties (e.g., centroids, moments of inertia, radius of gyration)</li> <li>G. Static friction</li> </ul>	
<b>9. Dynamics</b>	<b>7–11</b>
<ul style="list-style-type: none"> <li>A. Kinematics</li> <li>B. Linear motion (e.g., force, mass, acceleration)</li> <li>C. Angular motion (e.g., torque, inertia, acceleration)</li> <li>D. Mass moment of inertia</li> <li>E. Impulse and momentum (linear and angular)</li> <li>F. Work, energy, and power</li> <li>G. Dynamic friction</li> <li>H. Vibrations</li> </ul>	
<b>10. Strength of Materials</b>	<b>8–12</b>
<ul style="list-style-type: none"> <li>A. Stress types (e.g., normal, shear, bending, torsion)</li> <li>B. Combined stresses</li> <li>C. Stress and strain caused by axial loads, bending loads, torsion, or shear</li> <li>D. Shear and moment diagrams</li> <li>E. Analysis of beams, trusses, frames, and columns</li> <li>F. Deflection and deformations (e.g., axial, bending, torsion)</li> <li>G. Elastic and plastic deformation</li> <li>H. Failure theory and analysis (e.g., static/dynamic, creep, fatigue, fracture, buckling)</li> </ul>	

- 11. Materials Science** **6–9**
- A. Physical, mechanical, chemical, and electrical properties of ferrous metals
  - B. Physical, mechanical, chemical, and electrical properties of nonferrous metals
  - C. Physical, mechanical, chemical, and electrical properties of engineered materials (e.g., polymers, concrete, composites)
  - D. Corrosion mechanisms and control
- 12. Fluid Mechanics and Dynamics of Liquids** **8–12**
- A. Fluid properties (e.g., Newtonian, non-Newtonian)
  - B. Dimensionless numbers (e.g., Reynolds number, Froude number)
  - C. Laminar and turbulent flow
  - D. Fluid statics
  - E. Energy, impulse, and momentum equations (e.g., Bernoulli equation)
  - F. Pipe flow and friction losses (e.g., pipes, valves, fittings, Darcy-Weisbach equation, Hazen-Williams equation)
  - G. Open-channel flow (e.g., Manning equation, drag)
  - H. Fluid transport systems (e.g., series and parallel operations)
  - I. Flow measurement
  - J. Turbomachinery (e.g., pumps, turbines)
- 13. Fluid Mechanics and Dynamics of Gases** **4–6**
- A. Fluid properties (e.g., ideal and non-ideal gases)
  - B. Dimensionless numbers (e.g., Reynolds number, Mach number)
  - C. Laminar and turbulent flow
  - D. Fluid statics
  - E. Energy, impulse, and momentum equations
  - F. Duct and pipe flow and friction losses
  - G. Fluid transport systems (e.g., series and parallel operations)
  - H. Flow measurement
  - I. Turbomachinery (e.g., fans, compressors, turbines)
- 14. Electricity, Power, and Magnetism** **7–11**
- A. Electrical fundamentals (e.g., charge, current, voltage, resistance, power, energy)
  - B. Current and voltage laws (Kirchhoff, Ohm)
  - C. DC circuits
  - D. Equivalent circuits (series, parallel, Norton's theorem, Thevenin's theorem)
  - E. Capacitance and inductance
  - F. AC circuits (e.g., real and imaginary components, complex numbers, power factor, reactance and impedance)
  - G. Measuring devices (e.g., voltmeter, ammeter, wattmeter)

**15. Heat, Mass, and Energy Transfer**

**9–14**

- A. Energy, heat, and work
- B. Thermodynamic laws (e.g., 1st law, 2nd law)
- C. Thermodynamic equilibrium
- D. Thermodynamic properties (e.g., entropy, enthalpy, heat capacity)
- E. Thermodynamic processes (e.g., isothermal, adiabatic, reversible, irreversible)
- F. Mixtures of nonreactive gases
- G. Heat transfer (e.g., conduction, convection, and radiation)
- H. Mass and energy balances
- I. Property and phase diagrams (e.g., T-s, P-h)
- J. Phase equilibrium and phase change
- K. Combustion and combustion products (e.g., CO, CO<sub>2</sub>, NO<sub>x</sub>, ash, particulates)
- L. Psychrometrics (e.g., relative humidity, wet-bulb)